THE

THEORY AND PRACTICE

OF

LONGITUDE AT SEA:

COMPREHENDING

THE

THEORY of the SOLAR SYSTEM;
PHÆNOMENA of the MOON, other PLANETS and FIXED STARS;
DOCTRINE of PARALLAXES, TIME, and of the CELESTIAL SPHERE;
USE of NAUTICAL INSTRUMENTS;
New METHODS for LATITUDE by SUN, MOON and STARS;
LONGITUDE at SEA and LAND, by all the METHODS invented;
LATITUDE and LONGITUDE from COTEMPORARY OBSERVATIONS;

WITH OTHER
IMPORTANT IMPROVEMENTS.

BY SAMUEL DUNN,
Teacher of the Mathematical Sciences, LONDON.

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Enlarged, Improved, Corrected, and Revised, by the AUTHOR.

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TO THE HONOURABLE

THE

COURT OF DIRECTORS

OF THE

UNITED COMPANY

OF

MERCHANTS OF ENGLAND

Trading to the *EAST-INDIES*;

THIS WORK

(Comprehending a Variety of Improvements

MADE

UNDER THEIR PATRONAGE,

AND

Designed as a Course of Instructions for facilitating the Discovery of the LONGITUDE at SEA, to the Commanders and Officers of Ships in the COMPANY'S SERVICE)

With the greatest Deference and Respect, is

INSCRIBED,

By their most Obedient Humble Servant,

SAMUEL DUNN

PREFACE.

1. Navigation, Commerce and an Intercourse with Countries which are far distant from each other and separated by Oceans and Seas of great Extent, have been universally allowed to be so beneficial to Nations in general and to Individuals in particular; that for several Ages past to the present time, nautical Improvements have been admitted as being of the greatest Services to Maritime Kingdoms in general, and to the British Nation in particular.

2. The Machines whereby the Art of Navigation is performed are Ships, these may be considered as the most surprising and useful Instruments that have been invented, for they are driven by the Winds on the deep Waters of the Oceans and Seas, to the greatest Distances that can be measured round the Earth's Globe; their Navigators and Mariners conduct them by their Art, and readily discover the Places where they are at Sea, with the Courses and Distances to other Places or any designed Port.

3. Hence, Navigation depends on a Variety of Parts; first, on Naval Architecture or the Art of Ship-building, this gives the Form and Construction of the Ship with all its Parts, so that it may bear the Force of the Winds and Agitation of the Waters; secondly, the ordering and disposing of the Rigging, Tackle and other things in a proper manner, this is the Mariner's Art, and belongs to the working Seaman; thirdly, the guiding of Ships from one Place to another near the Coast, this is called Coasting, and requires great Care for avoiding Rocks, Shoals and other Dangers, but is otherwise assisted by Appearances of the Land and Marks near the Coast.

4. When a Voyage is making upon the Oceans and Seas of very great Extent, the Errors arising from Inability to steer the Ship the intended Courses, the Errors in the Distances measured, the Effects of Currents, and other Errors, will be such on the whole, as to deceive the Navigator in his Reckoning; and subject the Ship to Uncertainty and sometimes great Dangers.

5. At such perilous Times, and Places however far from Land, if the Ship's Latitude and Longitude can be taken, its Place on a Chart, with the Course to and Distance from other Places are shewn. This is the Perfection of Practical Navigation, it is that which Sovereigns and Princes have thought deserving their Munificence; Great Societies and Communities have encouraged Improvements in it, with Liberality and Rewards; Philosophers have examined every Part of Nature to find suitable Assistance for perfecting it from the Laws of Matter and Motion; Mathematicians have used all their Methods of reasoning and applied them with such Data as are applicable for this purpose; Astronomers have been employed in

obſerving the Motions of the Celeſtial Bodies, with Deſigns to make them ſubſervient to this end; Mechanics have exerted their greateſt Abilities, in conſtructing Timekeepers under various Forms and Names, to be aſſiſting in this Subject; and after all theſe, there are but five Methods whereby the Longitude at Sea may be taken, and but four of them generally practicable.

6. The firſt Method of finding the Longitude at Sea, depends wholly on Mathematical Science and ſuch Obſervations as are made in the Ship, concerning the Courſes, Diſtances and other things during the Voyage. When and where Errors ariſe in this Method, Aſtronomy gives it a great Correction, by the Application of an obſerved Latitude; and when or where the uſual Method of keeping a Reckoning at Sea will be attended with Errors, ſuch may be corrected, and frequently the Longitude had with Eaſe much more accurately therefrom, by the Method of Magnetic Sailing, which is explained in this Work.

7. The ſecond Method of finding the Longitude at Sea, depends partly on the Perfection of a Watch or Timekeeper, for keeping or meaſuring Equal Time, and partly on the Ability of the Navigator in making aſtronomical Obſervations, and comparing their Reſults with the Time ſhewn by the Watch. The aſtronomical Obſervations are eaſily to be made and computed; therefore, the principal part of this Method, depends on the Time-keeper. Its Uſes are here ſhewn in taking the Latitude in a Variety of new Methods.

8. The third Method of finding the Longitude at Sea, depends partly on the Accuracy of the Sextant or Octant, and partly on the Ability of the Navigators who make the Obſervations and Calculations. In this Method, the Aſtronomy of the Sun, Moon, and Fixed Stars, is now in a ſtate ſufficiently exact for the Longitude in all long Voyages; the Conſtruction of the Sextant and of the Auxiliary Tables, are ſufficiently exact for the Application of this Method, in long and important Voyages, and the Obſervations to be made, with the Calculations, are attended with no Difficulties but what can be eaſily overcome by the Perſons who obſerve or aſſiſt. This Method is amply treated of in the following Work.

9. The fourth Method of finding the Longitude at Sea, depends partly on the Laws of Nature, and partly on aſtronomical Obſervations; on the former for the Quantity and Quality of Magnetiſm, on the latter for making it applicable in Navigation.

10. The fifth Method is that by Jupiter's Satellites; this Method cannot be generally applied at Sea, by reaſon of the Ship's Motion and other Impediments which are herein mentioned.

11. The firſt Method was practiſed many Ages and (as may be ſuppoſed) in an imperfect manner, till about Two

hundred Years since, then it received its great Improvement by the Invention of the true Sea Chart. The second Method was known many Ages since, but could not be practised with Success till near One hundred Years since, then a Voyage was made near the Equinoctial, and Watches were found applicable for the Longitude at Sea; and since that they have been much improved. The third Method was known to the Ancients, but could not be practised by them for want of proper astronomical Tables for the Predictions, and proper Instruments for the Observations. The Sextant for taking angular Distances and the Lunar Tables for making Predictions were sufficiently exact for the Practise of this Method Fifty Years since, but the Problem itself (as it is now practised) has not been thought the best for this purpose, more than Thirty Years. The fourth Method was suggested One hundred Years since, but discontinued for want of knowing the Laws of the Magnetic System.

12. In order to make each of these Methods as useful as possible, the Navigator should be able to practise the first and third of them at least, and have the proper Instruments and Books for those Purposes; but, if the second Method be attended with unsuitable Expence, he may apply a common Pocket Watch for keeping short Intervals of Time, and in taking the Latitude according to the Directions in this Treatise. The Variation Method is attended with less Expence, and therefore there can be no Objection against it on that Account.

13. As Demonstrations and the best Reasons for things are most satisfactory to Persons of an inquisitive Mind, many may be desirous of such after seeing this Treatise. The Demonstrations relative to Arithmetic, Geometry, Trigonometry, Projection of the Sphere; the Rationale or Reasons for Operations in Plain Sailing, Mercator's Sailing, Middle Latitude Sailing, and great Circle Sailing; the Methods of Traverse Sailing, Oblique Sailing, Windward Sailing, Current Sailing; the Methods of taking the Latitude by Meridian Altitudes, and correcting the Longitude, may be seen largely explained from original Principles, in a Treatise by me on Practical Navigation. Since that Book was printed, the forementioned Method of Magnetic Sailing has been added to the usual Methods, and is easily to be applied with but little Deviation from the same Principles.

14. The Methods which are used in this Treatise for approximating the true Latitude, are proved to hold good by a Variety of Inductions; had either the Investigation or Demonstration of any one of these Methods been delivered alone, it might have been received by many Persons well inclined for Improvement as an unintelligible Jargon, never to be practised by them; but here, the Precepts are plainly expressed, the Ex-

amples

 PREFACE.

amples prove their Truth, and Persons of the slowest Apprehension may receive much more Assistance (if they require it) by using the Formulæ (or Formulas) invented by me for those Purposes.

15. Having the Latitude ready to be applied on all Occasions with Data that can be taken, the Longitude itself, the Variation of the Compass, and all other particulars arising in the apparent diurnal Motion, come of Course from easy Tables, and Operations never treated of by any other Person; this tends to the Discovery of Longitude at all times and places, independent of fallible Mechanism. By the same Data, without any previous Knowledge of the Latitude; both the Latitude and Longitude may be easily computed, by Persons who can work the Logarithms in the usual manner, altho' unacquainted with the Properties of the Sphere.

16. This Work likewise contains ample Instructions for knowing the Fixed Stars at all times and places, for distinguishing the Primary Planets and applying them; the Places of the principal Fixed Stars have been already settled by me for future Use, but the Places of the Primary Planets and those of the Moon, are to be taken from the proper Ephemerides, which are published periodically for shewing their daily Places in the Heavens.

17. The first Edition of this Work was written by me in a continued and popular manner; this second Edition is written in a synthetic way, in order to be more particular concerning the Meaning of the Definitions, Propositions, Theorems and Precepts, the Directions contained in them, and their being more easily referred to. The Sections alone amount to One hundred and fifty; the numbered Articles in them amount to Eight hundred and thirty, besides a great number of intermediate ones, and a plenty of Examples, which are general ones and not limited to any particular times or places, excepting such as require it from the nature of the Question.

18. Within Twelve Years last past, there have been Eight Treatises written by me, besides the present Edition of this Book and other Works; namely, Astronomy, Navigation, Longitude at Sea, Magnetic Variations, Latitude without Meridian Altitudes, Linear Tables for Longitude, Logarithms with Nautic Tables, and Courses of Formulæ for Latitude and Longitude at Sea. In these Works, the Reader will find the Principles and Practice of the Longitude at Sea, treated in a manner very different from that in which they have been pursued by other Writers on these Subjects, and at the same time carried to a greater height of Improvement, by easy Operations. In these things there was a necessity for Dissension from others; otherwise, it might have been unreasonable to expect Success in their Application at Sea. Twelve Copper-plate Prints accompany this Work.

THE

THE
THEORY AND PRACTICE
OF
LONGITUDE AT SEA, &c.

SECTION I.

Definitions and Principles, relative to the Diſcovery of the Longitude at Sea.

1. IN order to inveſtigate the Rules, Tables, and all other Requiſites whereby Obſervations and Calculations are made, previous to the Diſcovery of the Longitude at Sea; it is neceſſary to have a general and particular underſtanding of Arithmetic the Science of Numbers; Geometry that of Magnitudes; Trigonometry that of Triangles in all their kinds; Pneumatics and Hydroſtatics thoſe of the Air and Waters; Optics thoſe of Viſion and Light; the Properties of Matter, the Laws which it obſerves either when at Reſt or in Motion, its gravitating and centrifugal Forces; the Order, Dimenſions and Properties of the Celeſtial Bodies, their Diſtances and Motions in the Solar Syſtem; the Conſtruction and Application of ſuch Inſtruments, Tables and Principles as are applicable in making thoſe Diſcoveries; and the moſt unerring mathematical and phyſical Properties whereby they may be all applied, as occaſion requires, in deducing Concluſions concerning the true ſtate of the neareſt, intermediate and remoteſt Parts of the Univerſe or viſible World.

3. From theſe Principles and the Concluſions drawn from them; it follows, that the Earth on which we dwell would be nearly of a globular or ſpherical Form, if it could be perfectly at Reſt, but that by a Rotation round its Axis once in every Day (in a direction from Weſt toward Eaſt) its Diameter in that direction becomes lengthened and the Axis at right-angles thereto ſhortened, and that the whole Earth itſelf thereby becomes an oblate Spheroid flattened toward the Poles.

3. On this Figure of the Earth (by reaſon of Inequality in thoſe Diameters) in going equal Diſtances toward the ſame North or South Pole of the Earth, there becomes Inequality in either a Direction toward or from the Earth's Centre; and becauſe the Equality of Angles in either of thoſe Directions, regulates and expreſſes the angular Diſtances whereby the Degrees of Latitude are expreſſed, the Degrees of Latitude on the Earth's Surface, do thereby become ſhorteſt where that rotatory Motion is ſwifteſt, and lengthened in going toward either of the Earth's Poles. But,

4. Whatever the Difference is between a Degree of Latitude at either of thoſe extremes, in other words, either at the Poles or at the Equinoctial; near the Middle of the Quadrant, between the Equinoctial and either of the Poles, the length of a Degree of Latitude will be nearly a Medium between thoſe Extremes; and therefore, ſuch a Degree of Latitude may be taken for a mean Degree of Diſtance, and its ſixtieth part a geographical or nautical Mile; and ſuch a Meaſure may not be improperly applied in expreſſing the Diſtances of the Planets from the Sun, and the Dimenſions of their Orbits in the Heavens. Such is the Meaſure of a Mile in this Work.

II.

Of the Diurnal Motion or Daily Rotation of the Earth, round its own Axis.

1. As the Sun, Moon and Stars are obſerved to riſe, appear, ſet and diſappear at many places of the Earth, in the interval of either a little more or leſs than a natural Day; it is therefore concluded that, either thoſe Bodies muſt move round the Earth in that interval, or on the contrary that the Earth's Ball itſelf muſt turn round its own Axis in the ſame time. The Abſurdities that would follow on a ſuppoſition of the firſt of theſe, have cauſed it to be exploded, and the ſtrict agreement of the latter with all the eſtabliſhed Laws of Nature and Phænomena of the Univerſe, have cauſed it to be generally received.

2. This rotation gives exiſtence to the firſt Definitions concerning Latitude and Longitude on the Earth's Surface; and the Diſcovery of theſe from actual Obſervations becomes eaſily applicable for determining the Situations and Diſtances of Places, both on Land and at Sea. When ſuch Situations and Diſtances can be taken

taken on Land, they are applicable for Improvements in Geography; and when they can be taken at Sea, the Place of the Ship may be thereby readily determined, and this tends to the Perfection of Practical Navigation.

3. In Figure 1. Let *a* represent the Centre of the Earth, and *bc* its Polar Axis; *bdce* a great Circle passing through the Poles; *dae* another great Circle, every where equidistant from the Poles, and *blhkg* another great Circle passing through the Poles.

4. Round the immoveable Axis *bc*, whilst it is parallel to the Earth's Axis, let the Globe turn from West toward East, with an equable Motion so as to make one Revolution in a Day, and it will represent the Earth's diurnal Rotation, whereby the Celestial Bodies appear as though they did move round the Earth's Axis from East to West in the same time.

5. The Points *b* and *c*, represent the Poles of the Earth. Great Circles passing through the Poles and the Equinoctial are Meridians on the Earth. From the Equinoctial toward either of the Poles, the Latitude of a Place is reckoned, in Degrees and Minutes, both northward and southward.

6. For the Longitude either of a Place on the Earth, or a Ship at Sea, from London; let *l* be London, and either *f*, *h*, *k*, *g*, or *i*, the Place on Land, or Ship at Sea; then, the Distance *dg* is the Measure of either the Angle *lbf*, or *lbh*, or *lbk*, or *dci*, in Degrees and Minutes, and it is the Longitude from London, East when a Person is East; and West when a Person is West of the Meridian of London.

III.

Of Horizontal Parallaxes in general, and the Moon's Horizontal Parallax in particular.

1. The Zenith is that Point in the Heavens which is perpendicularly over a person, wherever he is.

2. The Nadir is that Point in the Heavens which is perpendicularly under a person, wherever he is.

3. The True Horizon of a place is a great Circle whose Plane passeth through the Earth's Centre, and every part of it is Ninety Degrees distant from the Zenith.

4. The Apparent or Visible Horizon is nearly a great Circle, whose Plane passeth through a Place on the Earth's Surface, and its Circumference is nearer to the Zenith than the true Horizon by a Semidiameter of the Earth.

5. In either taking or expressing the Altitude of any Celestial Body, it is the Elevation above the apparent Horizon when that Horizon is not encumbered with objects that take off the Regularity of the Earth's Figure; where such happen on Land, the apparent Horizon is in the Tangent to the point that would be there drawn to the Earth's Surface.

6. As the Earth's Figure is that of an Oblate Spheroid flattened toward the Poles, its Equinoctial thereby becomes a Circle whose Circumference is somewhat greater than that of an elliptic Meridian; therefore, in going toward the Earth's Poles, the difference between the true and apparent Horizon in a north and south direction, will not be the same as at the Equinoctial, and this difference will be more various although less in quantity under other horizontal Positions; but it belongs to the greatest refinements in Science to assign what its exact quantity is, and in usual cases, that for the Equinoctial is generally taken and applied.

7. Consequently, when a Celestial Body is at a distance from the Earth which is indefinitely great, the true and apparent Horizons will coincide, and therefore, in such cases, it makes no difference which of the two is either meant or used. But,

8. When a Celestial Body is so near to the Earth, that the Earth's Semidiameter bears any sensible or perceptible Ratio to the Distance of the Body from the Earth, the true and apparent Horizons will then differ from each other, by a sensible or perceptible Angle.

9. The Difference between the true and apparent Horizons is the Horizontal Parallax; and this is the Angle which the Semidiameter of the Earth would appear to subtend to the Eye of an Observer, could it be placed at the Centre of the Celestial Body; therefore, both of these ways will express the Horizontal Parallax.

10. Consequently, the Earth's Semidiameter being known, and it being also known that many of the Celestial Bodies are too far distant to bear any sensible Proportion to the Semidiameter of the Earth, the Horizontal Parallaxes of such are imperceptible. Hence, the Moon's Horizontal Parallax, is greatest, because she is the nearest Planet to the Earth; but the Horizontal Parallax of Saturn is a much smaller Angle, especially when at his greatest Distance from the Earth; and the Horizontal Parallax of a Fixed Star is nothing.

IV.

Of the Earth's Orbit, and the Annual Motion of the Earth round the Sun.

1. As the Earth's daily Rotation round its own Axis, solves the daily apparent rising and setting of

of the Celeſtial Bodies, ſo the annual Motion of the Earth round the Sun, ſolves the various Poſitions of the Sun, either at Noon, or any other Time of the Day, from one Day to another throughout the Year; and the like for the Stars.

2. The Interval in which the Earth thus performs one whole Revolution round the Sun is a Year; and whilſt it is thus moving, it is kept in its Orbit by two Forces or Powers, the firſt of which is centrifugal, whereby it is continually endeavouring to fly off from its Orbit, in the direction of a Tangent to it; the ſecond is a gravitating Force, whereby it is continually tending toward that common Centre of Gravity which is near the Sun.

3. At firſt ſight, it ſeems as though things would be more ſimple if the Earth and other Planets moved in Circles; but, it is well known that ſuch Motions of Bodies round a Centre, in free Space, cannot ariſe from a Compoſition of the centrifugal and gravitating Forces; and that contrarily, elliptic Orbits become formed, having oppoſite Points, one nearer to, the other farther from the Centre, than the mean Diſtances.

4. In Figure 2. Let a be the Centre of the Circle $bdce$, b the Earth, ab the Sun's Diſtance from the Earth 83970000 Miles; make as the Eccentricity 141080 Miles; let s be the Focus of an Ellipſis whoſe Tranſverſe Diameter is bc; Conjugate Diameter fg; and whoſe elliptic Circumference is $bhickgl$.

5. Let s repreſent the common Centre of Gravity of the whole Solar Syſtem, and near the Sun, whoſe large Body is continually gyrating round that Centre by the Laws of the Solar Syſtem; the two Centres being always within the diſtance of the Sun's Semidiameter from each other.

6. Let the Earth be either at b, h, f, i, c, k or l, moving onward in its Orbit, ſo as to perform one whole Revolution round the Sun in a Year. Then, in this annual Orbit,

7. The Perihelion point or neareſt approach of the Earth to the Sun, will be at b. The Aphelion point or fartheſt receſſion of the Earth from the Sun, will be at c. When the Earth is either at f or g, they are places of Mean Diſtance from the Sun. Farther,

8. Whilſt the Earth is moving from its Aphelion toward its Perihelion, it moves more ſwift; but, in moving from its Perihelion toward its Aphelion more ſlow, and in ſuch manner, that the Areas between the Arches deſcribed and the Radii drawn to the Sun, are very nearly proportional to the Intervals of Time in which thoſe Arches are deſcribed.

9. This irregular Motion is the natural conſequence of the gravitating and centrifugal Forces acting contrary to each other, under oblique Directions; hence, theſe are the means of preſerving the Earth in its annual Orbit; for, did the gravitating Force act alone, it would carry the Earth to the Sun; and, did the centrifugal Force act alone, it would carry the Earth to remote Regions, which, to the Inhabitants of this World are entirely unknown.

V.

Of the Moon's Orbit, and the monthly Motion of the Moon round the Earth.

1. As the Earth is carried round the Sun in the interval of a Year, ſo is the Moon carried round the Earth in the interval of a Lunar Month, in an Orbit which has a tendency to be nearly elliptical, and which is formed by the ſame Cauſes and the ſame Laws; but with theſe differences, the place where the Moon becomes neareſt to the Earth is the Perigee; the place where the Moon becomes fartheſt from the Earth is the Apogee.

2. In Figure 3. Let p be the Earth's Perihelion, a the Aphelion, and h a point in the Earth's Orbit where the Earth is, in moving round the Sun.

3. At h with an Eccentricity of 17000 Miles, and a mean Diſtance of 241000 Miles, draw the elliptic Orbit $ilkm$, repreſenting the Moon's Orbit deſcribed round the Earth in a Month.

4. Let i be the place of the Moon's neareſt approach to the Earth in that Orbit, and it is the Perigee; k the fartheſt receſſion, and it is its Apogee; becauſe s is the Centre of the Earth's Orbit, the Earth is called a Primary Planet; becauſe h is the Centre of the Moon's Orbit, the Moon is called a Secondary Planet.

5. Let the Moon be either at l, m, or any other place in her Orbit; then, by the Law of Gravity ſhe will tend toward the Sun in the Direction ls, or ms, but toward the Earth, with a greater Force, on account of the nearer diſtance in the direction lh or mh. The quantities expreſſing theſe two Forces reſpectively, together with ſome others of leſſer conſequence, muſt be found for all parts of both the Earth and Moon's Orbits, before the Moon's place in her Orbit can be accurately predicted.

6. It is farther to be noted that, whilſt the Moon is making the periodic Revolution, the Earth itſelf is removed by its Motion near a Degree forward in its Orbit, and that on account of the Earth's ſwift Motion compared with that of the Moon, inſtead of an elliptical Lunar Orbit, that Orbit becomes wholly concave toward the Sun; and the Moon's place therein becomes regulated by nearly the ſame Principles.

VI.

VI.

Of Time and the Equation of Time, as derived from the annual Motion of the Earth, and the Positions of the Celestial Bodies.

1. It having been long since, and to the present Age found that, mechanical Methods carefully executed, are insufficient for measuring accurately small and large Portions of Time; the performance of this most difficult Task has therefore been thought possible, by no other Method but by astronomical Observations, and such predictions as may be derived from them, relative to the apparent Places of the Sun and other Phænomena in the Heavens.

2. When Time is derived from the apparent Places of the Sun, it is therefore called Solar Time; when from the apparent Places of the Fixed Stars, it is called Siderial Time. When Portions of Time are made up of equal Portions of Duration, the Time is said to be Equal Time; but when such Portions of Duration are unequal, the Time made up by them is said to be unequal.

3. The manner of determining the quantities of Time by the Motion of the Earth round the Sun is thus. In Figure 4. Let *b* represent the Centre of the Sun; *a* the Centre of the Earth; *d* its north Pole; *c* its south Pole, *dlfhcgei* a Meridian on the Earth, and *man* an Arch of the Earth's Orbit.

4. Let *id* be 66 degrees and 32 minutes, the Arch which the north Pole of the Earth is elevated above the Plane of the Earth's Orbit, whilst the Earth is moving round the Sun.

5. Near the Beginning of the Year, wherever a person is situated on the Earth, let the Diameter of the Sun be carefully measured, making correct allowance for Refraction; also, near the Middle of the Year, let that Diameter be measured, and the former will be 64 seconds of a Degree more than the latter.

6. Consequently, the annual Orbit described by the Earth is not circular, and the Earth is in Perihelion in our Winter, but in Aphelion in our Summer; and by the Laws of Nature the Motion is swiftest in Perihelion; therefore the Winter Half Year is shorter with us than the Summer Half Year, first because of the Sun's Eccentricity, and secondly on account of Inequality in the Motion. But farther, it is found that the Earth's Axis from Pole to Pole, keeps very nearly one and the same Parallel Position, according to the Observations of Astronomers.

7. From these principles, by reason of the Earth's equable Rotation round its own Axis, the constant Parallelism of that Axis, and the vast distance of the Fixed Stars from the Solar System, the Siderial Days are equal in length, and there is the same length of Duration from the instant a Fixed Star appears on the Meridian, to its Return to it, at all places and at all times of the Year.

8. The Solar Day at a place, suppose for instance at London, begins when the Centre of the Sun is apparently on the Meridian of London, and ends when that Centre returns apparently to the same Meridian. The like for the Meridians of other places.

9. Consequently, the Solar Day is unequal on two accounts; first by reason of Inequality in the Earth's Motion through its Orbit, and secondly, through a continued change of Position, between the Earth's Axis and that part of the Orbit where the Earth is.

10 Therefore, the same Fixed Star appears to come to the Meridian of a place, some times a little more and at other times a little less than 3 m. 57 s. of Time every Day, sooner than the Sun.

11. The Solar Day is continually varying its length at all times of the Year; and has its Noon sometimes a little sooner, at other times a little later than the preceding Day.

12. A Mean Solar Day, is that of unequal length from the Beginning to the End of the Solar Year.

13. The Difference between the Solar Day and the Mean Solar Day, is the Equation of Time. This is commonly expressed in a Table for every Day in the Year, and called a Table of the Equation of Time.

14. A well-regulated Pendulum Clock, carefully compared with Meridian Transits of the Fixed Stars, will shew nearly the Change or Variation from Siderial to Solar or Mean Solar Time, from one Day to another, and so throughout the Year. But,

15. Should there happen to be any hidden Cause whereby the equable Rotation of the Earth is disturbed, or the constant Parallelism of its Axis varied, or any unequal Measurement of Time by the Clock; the Accuracy would be lost.

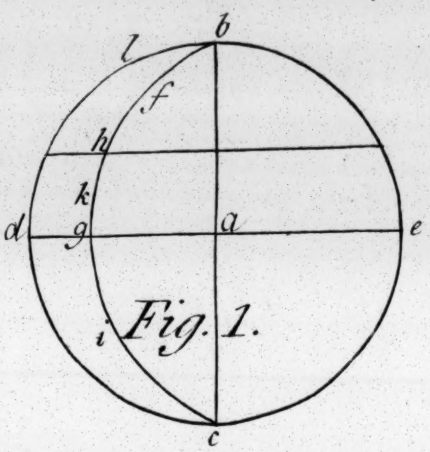
16. It is most certain to derive the Differences or Variations from either Siderial or Mean Solar Time, which form an Equation Table; from the Laws of the Solar System, and from proper Observations.

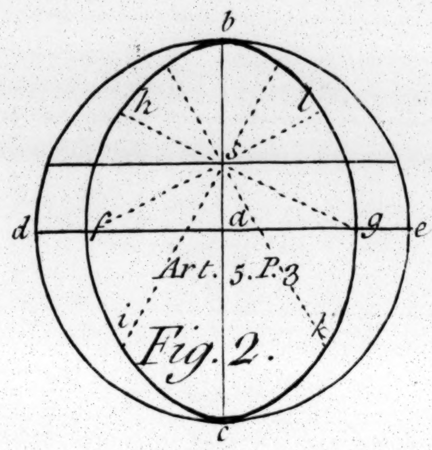
17. In this Method; should there be any Error in taking the Meridian Transit by the Clock; or any Inequality in the Diurnal Rotation of the Earth; or any unusual Motion of the Earth in its Orbit; or any Change in the Position of the Earth's Axis; in either of these Cases, it may be attended with the greatest Difficulties to seek the exact quantity of the Equation of Time.

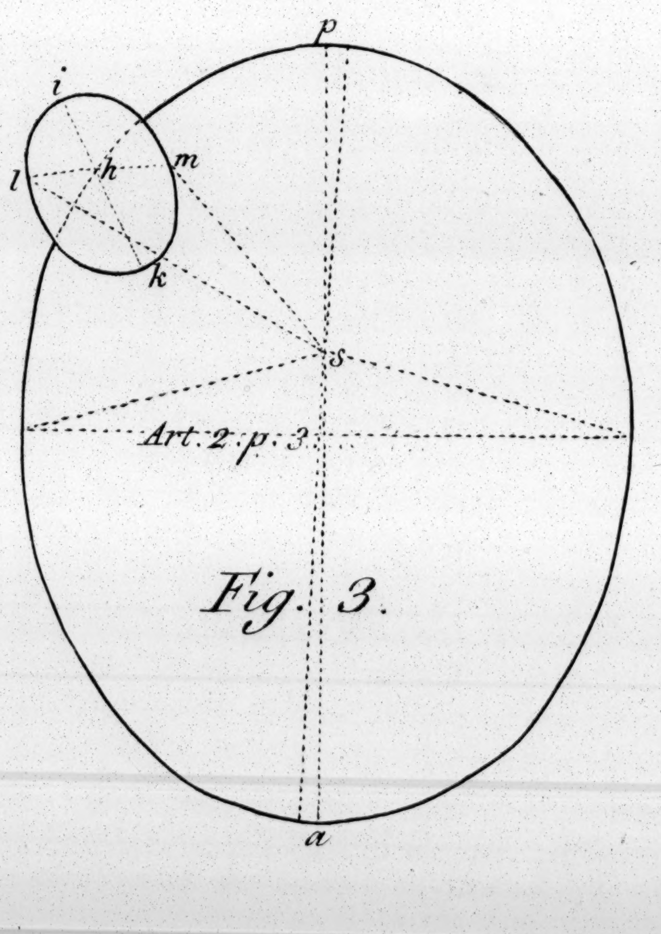
18 When

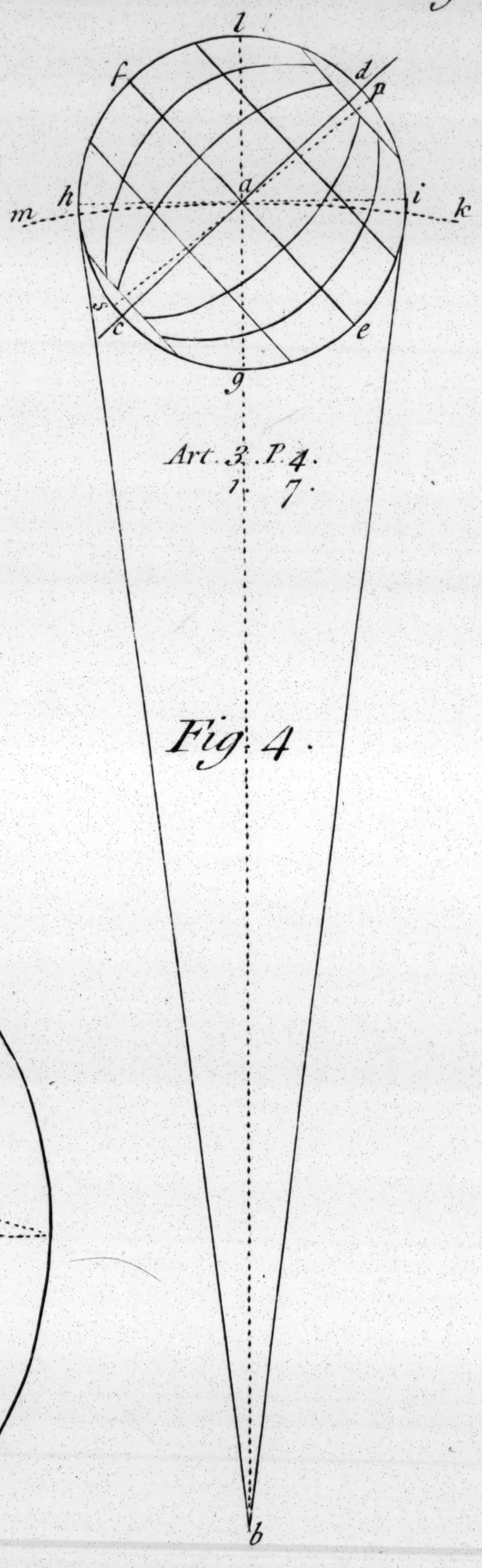
Motion of Earth and Moon.

Plate 1.
Page 4.









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18. When proper Aſtronomical Obſervations have been made, either on Land or at Sea, Solar Time is that which is there immediately deduced from obſerved Altitudes of the Sun; on Land it is readily had from obſerved Meridian Tranſits of the Sun, and it is the Time which Aſtronomers in all countries make uſe of in predicting Phænomena.

19. When the Plane of the Meridian of any place either on Land or at Sea, being produced, paſſeth through the Centre of the Sun, the Solar Day begins and the former Day ends, in the neareſt Semicircle of that Meridian; and therefore the ſame quantity of the Equation of Time, is applicable for all parts of that Semicircle, from the Equinoctial Line to the Poles. But,

20. Becauſe there is near Half a Minute of Time Difference in the Equation from one Day to another about the 20th of December, when an Equation Table is made for one Meridian and a Ship or Perſon is at another, there muſt be a proportional Allowance when ſuch or leſſer Differences happen, in applying the Equation to a diſtant Meridian, or there will be an Error proportional to that Diſtance and the Daily Difference of the Equation.

VII.

Of the Solſtices and Colures.

1. To Inhabitants on the North Side of the Equinoctial, the Summer Solſtice is when the Sun appears to have made his neareſt approach to the North Pole, or rather to the north Part of the Earth's Axis produced to the Heavens.

2. The Winter Solſtice is when the contrary circumſtances happen, Half a Year after.

3. To Inhabitants in North Latitude, the Summer Solſtice is when the Earth comes to ſuch a place in its Orbit, that a produced Plane perpendicular to the Plane of the Orbit, paſſeth through the Poles of the Earth. This happens near the 20th of June.

4. At the Winter Solſtice, that Plane and the Poles coincide in an oppoſite or contrary Poſition to the former; this happens near the 20th of December.

5. The times of the Solſtices cauſe the longeſt and ſhorteſt Days reſpectively.

6. Therefore, the Summer and Winter Solſtice are in one circular Meridian, or rather in two oppoſite Semicircular Ones, in one of which is the Sun's greateſt apparent approach to the North, and oppoſite to it the greateſt approach to the South Pole.

7. When the Earth comes to a place in her Orbit where the Plane of the Earth's Equinoctial produced paſſeth through the Centre of the Sun, the Earth's Poles are equally illuminated by the Sun, and the great Circle paſſing through the Sun and the Earth's Axis produced, is the Equinoctial Colure.

8. When the Sun is apparently in the Equinoctial Colure near the 20th of March, it is ſaid to be the Time of the Vernal Equinox; when the like near the 22d of September, the time of the Autumnal Equinox, to Inhabitants in North Latitude. The contrary for South Latitude.

VIII.

Of the Ecliptic and Zodiac.

1. Through the Points where the Sun apparently is reſpectively at the Winter Solſtice, at the Perihelion, at the Vernal Equinox, at the Summer Solſtice, at the Aphelion, and at the Autumnal Equinox, with proper Tranſverſe Diameter, Conjugate Diameter and Eccentricity, draw the Ellipſis and it is nearly the Earth's Orbit deſcribed in a Year. See Figure 3.

2. Transfer this Orbit to the Celeſtial Sphere of the Heavens, and it will naturally follow from the Inequality of Central Forces, that the Earth moves faſteſt near the Perihelion, ſloweſt near the Aphelion, and that therefore no equal Diviſions in an Orbit apparently circular, can adequately repreſent the true Subdiviſions of this Orbit.

3. Hence it appears, that, Inſtruments and Models, however expenſively finiſhed, upon Circular Diviſions and Subdiviſions; can but imperfectly repreſent the Inequalities which ariſe from the Laws of the Solar Syſtem, and which will be found, even amongſt theſe parts of the Celeſtial Motions.

4. The Zodiac is a Zone or Belt of near 17 Degrees in Breadth, extending round the Heavens, Half this Breadth being on the North and the other Half on the South of the Ecliptic Line.

IX.

Of the Celeſtial Meridians and Equator.

1. Produce the Earth's Axis both ways to an indefinite diſtance in the Heavens, and their Ends are the Poles of the Equator.

2. Suppoſe an indefinite number of Great Circles drawn ſo as to paſs through the Poles of the Equator, and they are the Celeſtial Meridians.

3. The Equator is a Great Circle whoſe Plane is at Right Angles to the Earth's Axis, and its Parts are equally diſtant from its Celeſtial Poles.

4. The Meridians and the Equator are each of them ſuppoſedly divided and ſubdivided into equal Degrees, Minutes and Seconds; theſe Diviſions ariſe from the ſuppoſed perfect Equability of the Earth's Rotation.

5. The Angle which the Plane of the Equator makes with the Plane of the Ecliptic is at present 23° 28′ nearly, but the Angle made by the Equator and Meridians is always a Right-Angle. Consequently, a Right-angled Spherical Triangle is formed by a part of the Ecliptic, a part of a Meridian, and a part of the Equator; the first of these expressing the Sun's Longitude, the second his Declination, and the third his Right Ascension.

X.

Of the Ecliptic and Equator.

1. The Circumference of the Ecliptic Line contains 360 Degrees, these Degrees are unequal as arising from an unequal Motion of the Earth round the Sun, but equal when corrected and arranged in Tables, so as to compare with the Circles of the Celestial Sphere. Then,

2. The Ecliptic contains the Twelve Signs; Aries, Taurus, Gemini, Cancer, Leo, Virgo, which are northern Signs; Libra, Scorpio, Sagitarius, Capricornus, Aquarius, Pisces, which are southern Signs.

3. The Ecliptic and Equator both begin at the Equinoctial Point Aries; its first Quarter ends at the solsticial Point Cancer; its middle is at the equinoctial point Libra, and its third quarter ends at the solsticial point Capricorn, and each of these Signs contains Thirty Degrees. The Equator is numbered quite round to 360 Degrees.

XI.

The Dimensions of the Solar System.

1. The Solar System, is generally understood to consist of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, the Moon, the four Satellites of Jupiter and the five Satellites of Saturn, with Saturn's Ring. See Figure Solar System.

2. The Common Centre of Gravity in this System, is within the Sun's Surface, and the Sun's Centre gyrates round that Centre of Gravity, whilst the Centre of Gravity is either at Rest or is moving toward other Systems.

3. The Primary Planets respect the Centre of the Sun nearly, their Orbits round him are nearly elliptical; their Mean Distances from the Sun, Periodic Times, Ascending Nodes, Aphelions, and Inclinations to the Plane of the Earth's Orbit, are thus:

Orbit of *Mercury.*

Mean Distance	32504700 Miles.
Periodic Time	87d. 23h. 15m.
Asc. Node	1s. 15d. 47m.
Aphelion	8s. 14d. 14m.
Inclination	6d. 59m.

Orbit of *Venus.*

Mean Distance	60738200 Miles.
Periodic Time	224d. 16h. 41m.
Asc. Node	2s. 14d. 42m.
Aphelion	10s. 7d. 51m.
Inclination	3d. 23m.

Orbit of *Mars.*

Mean Distance	127938600 Miles.
Periodic Time	686d. 22h. 18m.
Asc. Node	1s. 18d. 19m.
Aphelion	5s. 2d. 12m.
Inclination	1d. 51m.

Orbit of *Jupiter.*

Mean Distance	436726500 Miles.
Periodic Time	4330d. 8h. 35m.
Asc. Node	3s. 8d. 45m.
Aphelion	6s. 11d. 7m.
Inclination	1d. 19m.

Orbit of *Saturn.*

Mean Distance	801021000 Miles.
Periodic Time	10750d. 13h. 15m.
Asc. Node	3s. 21d. 46m.
Aphelion	9s. 0d. 27m.
Inclination	2d. 30m.

4. In these Orbits, the Nodes and Aphelions are not perfectly at Rest, but affected with an alteration of a few Seconds per Year.

5. In these Orbits, the greatest Horizontal Parallax is that of Venus in Perigee, which is 31 Seconds; Mars in Perigee gives but 16 Seconds, the others are either smaller or nothing.

6. The Moon as a Secondary Planet respects the Earth for her Centre, but is nevertheless solicited toward the Sun. The Mean Inclination of her Orbit to the Plane of the Earth's Orbit is 5d. 9m. her Mean Distance from the Earth 232440 Nautical Miles. She always presents nearly the same Face toward the Earth; and apparently moves round the Earth so as to come up with the Sun, in 29d. 12h. 44m.

7. By presenting the same Face, she revolves round her own Axis, in the Time of one Revolution round the Earth; and librates both from East to West and from West to East, from North to South and from South to North, whereby the Spots near her Limbs alternatively appear and disappear to Observers on the Earth.

8. The four Satellites of Jupiter move round him in these Periodic Times; the

First in	1d. 18h. 28m.
Second in	3d. 13h. 16m.
Third in	7d. 4h. 0m.
Fourth in	16d. 18h. 5m.

9. The

9. The five Satellites of Saturn move round him in these Periodic Times; the

First in	1d.	21h.	18m.
Second in	2d.	17h.	41m.
Third in	4d.	12h.	25m.
Fourth in	15d.	22h.	41m.
Fifth in	79d.	7h.	41m.

10. The Diameters of the Sun and Planets in Nautical and English Miles, are

	Nautic Miles.	Eng. Miles.
Sun	775548.	896727.
Mercury	2831.	3273.
Venus	6683.	7727.
Earth	6876.	7950.
Moon	1877.	2170.
Mars	4611.	5281.
Jupiter	78338.	90578.
Saturn	69448.	80297.

11. The Angles which the Diameters of the Sun and Planets subtend at their Mean Distances of the Sun from the Earth and Planets, are

Sun, from Earth	32m.	6s.
Mercury, from Sun	0m.	7s.
Venus, from Sun	0m.	$16\frac{1}{2}$s.
Earth, from Sun	0m.	17s.
Moon, from Sun	0m.	5s.
Mars, from Sun	0m.	$11\frac{1}{2}$s.
Jupiter, from Sun	3m.	14s.
Saturn, from Sun	2m.	52s.
Saturn's Ring	6m.	41s.

12. Their Heliocentric Places are as they would be seen from the Sun; their Geocentric Places as they are seen from the Earth.

XII.

Of Latitude, Longitude, Declination and Right Ascension, in the Heavens.

1. The Poles of the Ecliptic are each 90 Degrees distant from every part of the Ecliptic Line, north and south, in the Surface of the Celestial Sphere; these are in the Circles of Latitude, and the Latitude is reckoned from the Ecliptic.

2. The first Circle of Latitude passeth through the Beginning of the Ecliptic. Other Circles of latitude are supposedly drawn from the nearest Poles, through the Celestial Body to the Ecliptic, and the Ecliptic Arch intercepted from its Beginning is the Longitude.

3. Declination is the Distance on a Meridian, from the Equator; Right Ascension is the Distance from that point of the Equator, to the Beginning of the Ecliptic.

4 Hence, the Sun has Longitude, Right Ascension and Declination, but no Latitude. All the Planets may have Latitude, Longitude, Right Ascension and Declination, and so have all the Fixed Stars, except what are in the Ecliptic Line, these have no Latitude.

XIII.

The Causes of Recession at the Equinoctial Points.

1. Let *b*, Figure 4, be the Centre of the Sun, *a* the Centre of the Earth, *ae* the Semi-equatorial Diameter and *ac* the Semi-polar Axis; so is *ce* a Quadrant of the Elliptic Meridian, in which let *gc* and *ge* be equal, then will *eb* be shorter than *cb*, and consequently, by the Laws of Gravity, the Earth's Axis *cd* will thereby take another Position as *sp*, and all the parts of the Earth therewith.

2. An effect of the same kind, ariseth from the Moon's Motion round the Earth; but greater on account of her being nearer to the Earth.

3. By these causes and effects; when the Earth is at that place in its Orbit, where the Equinoctial Point would be, if the Earth was a perfect Sphere and had equable Parallelism, the small Change of Parallelism, throws that Point 50 Seconds per Year westward from the Beginning of the Signs.

XIV.

Of the Revolution of the Ecliptic Poles, and yearly Precession of the Fixed Stars in Longitude.

1. By the Recession of the Equinoctial Points from East toward West, the Poles of the Ecliptic move the same Way parallel to the Ecliptic at the Rate of 50 Seconds and a third, of Longitude per Year.

2. Consequently, every Fixed Star gains so much in Longitude yearly, or a Degree of Longitude in $71\frac{1}{2}$ Years, whilst its Latitude remains the same.

3. All the Fixed Stars will thereby be carried round the Poles of the Equator from West to East in 25750 Years, and in that Time, different Stars will become Pole Stars for the north and south Equatorial Poles, if the World continues so long.

4. By the same principle, different Stars have been Pole Stars since the Creation.

5. Hence, the Images of the Constellations and the Stars in them, are removed a Sign forward from the zodiacal and other divisions of the same Names, through length of Time.

XV.

Of Changes in the Moon's Orbit; her different Horizontal Parallaxes and apparent Diameters.

1. The Inclination of the Moon's Orbit to the Plane of the Ecliptic is never less than 5d. 0m, nor more than 5d. 17m. Consequently, the Poles

of that Orbit describe a Curve round the Poles of the Ecliptic, whose greatest Breadth is 17m.

2. As the Equinoctial Points have a Recession contrary to the order of the Signs, so have the Moon's Nodes a Recession the same way from similar causes, but upon the whole much swifter, although sometimes they are almost at rest; this carries them quite round the Orbit in somewhat less than 19 Years, so that afterward the New and Full Moons return, nearly as before.

3. The Moon's Mean Motion in her Orbit is 13d. 10m. 35s. per Day, this carries her round her Orbit in 27d. 7h. 43m. She comes to the Sun in 29d. 12h. 44m. She revolves to her Ascending Node in 27d. 5h. $5\frac{1}{2}$m. and to her Apogee in 27d. 13h. $18\frac{1}{2}$m.

4. The Periodic Times in which the Moon's Apogee and Node make one whole Revolution round the Lunar Orbit, and come up with the Sun or Stars, are

Apogee to Sun	3231d.	8h.	35m.
Apogee to Star	3232d.	11h.	$14\frac{1}{2}$m.
Node to Sun	6798d.	4h.	53m.
Node to Star	6803d.	2h.	$55\frac{1}{4}$m.

5. Hence, the Moon's Apogee revolves quite round her Orbit in 8y. 309d. 8h. 20m. and her Node moves quite round her Orbit in 18y. 224d. 5h.

6. The Moon's greatest and least Horizontal Parallaxes arise from the Eccentricity of the Lunar Orbit and the Earth's Semidiameter. Hence the

Greatest Hor. Parallax	61m.	35s.
Least Hor. Parallax	54m.	6s.
Greatest Diameter	33m.	34s.
Least Diameter	29m.	29s.
Greatest Semidiam.	16m.	47s.
Least Semidiam.	14m.	45s.

7. The Sine of the Mean Horizontal Parallax is to Radius, nearly as the Earth's Semidiameter is to the Moon's mean Distance from the Centre of the Earth.

8. The Moon's Diameter may be taken, either on Land or at Sea; and then it will be, As her greatest Semidiameter, to her greatest Horizontal Parallax; so is her observed Semidiameter, to her present Horizontal Parallax. Each in Seconds.

XVI.

Of Jupiter's Satellites.

1. These Secondary Planets have four different Situations which are observable to Spectators on the Earth; first, by disappearing in his Shadow; secondly, by disappearing behind his Body; thirdly, by almost disappearing in passing between him and the Earth; fourthly, by passing apparently near to one another.

2. Of these; the first and fourth Circumstances are the most instantaneous Phænomena, and the first Satellite immerges quickest into, and emerges quickest from the Shadow.

3. The Immersions may be observed from the Opposition to near the Conjunction; Emersions from the Conjunction to near the Opposition; therefore, for the Immersions Jupiter should be East, and for the Emersions he should be West of the Sun.

4. Correspondent observations of either of these particulars, may be applied as instantaneous Phænomena; such may be easily made on Land by help of good Telescopes, but at Sea it cannot be practised under any considerable Motion of the Ship.

XVII.

Of Saturn's Satellites.

1. The vast Distance of Saturn from the Sun, renders his Satellites at all times difficult to be observed, without the best Telescopes.

2. The irregular Positions in which they appear, renders them difficult to be observed and rightly distinguished.

3. In some Positions of Saturn, these Satellites have their Eclipses, Occultations, and Transits not much unlike the Satellites of Jupiter, in others very different; otherwise more Utility might arise from observing them.

XVIII.

Of the Sun's Separation from the Fixed Stars, in Right Ascension, as seen on Land and at Sea.

1. The Fixed Stars keep near the same places for a considerable time, except the Correction which arises from the Recession of the Equinoctial Points.

2. Mean Solar Time is that which for a Year measures the same number of equal Days, Hours, Minutes and Seconds; as are in a Year of unequal Solar Time, measured by the Sun.

3. Because the Earth moves swiftest in Perihelion and slowest in Aphelion, the former alters its Right Ascension 4m. 27s. of Time per Day near the Winter Solstice, and the latter alters its Right Ascension 4m. 10s. of Time per Day; the Medium is 4m. 18s. per Day for the Mean Meridian Transits of the Fixed Stars sooner than the foregoing Day.

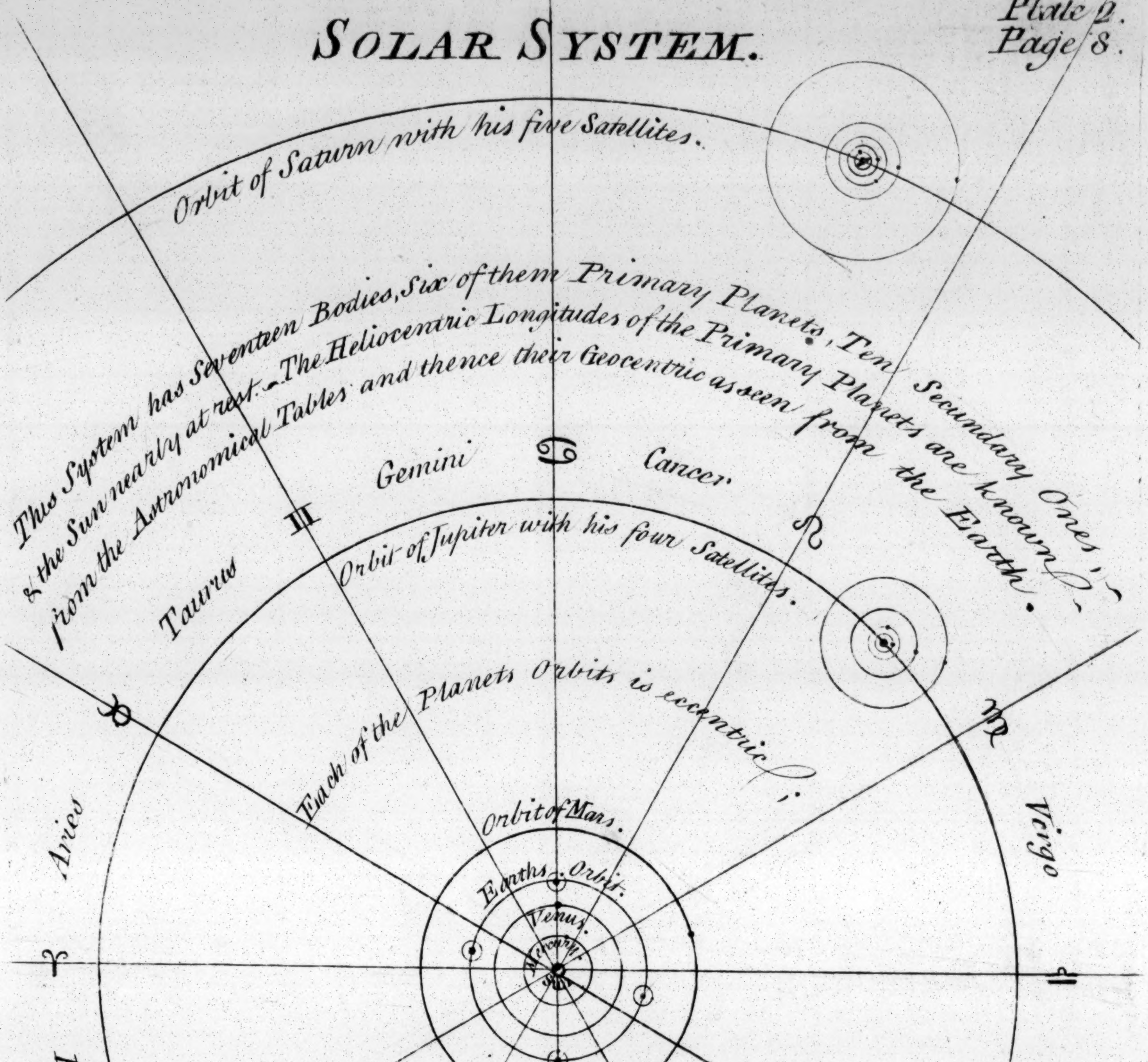
XIX.

Of the Sun's Separation from the Fixed Stars in Longitude, as seen on Land and at Sea.

1. Because the Sun apparently moves along the Ecliptic Line, and apparently swifter when the Earth is in Perihelion; therefore at that time the Fixed Stars separate from the Sun in Longitude 1d. 1m. 11s. per Day, and in Aphelion 0d. 57m. 10s. per Day; so the Mean is 0d. 59m. 10s. per Day.

SOLAR SYSTEM.

Plate 2.
Page 8.



The Sun's Mean Distance from the Earth is near 90 Million Miles.

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XX.

Of the Primary Planets Separation from the Sun in Right Aſcenſion and in Longitude, as ſeen on Land and at Sea.

1. The Right Aſcenſions ariſe from the Inclinations of their Orbits to the Plane of the Earth's Orbit, and from the Heliocentric Longitudes.

2. Mercury, Venus, Mars, Jupiter and Saturn, do each of them at different times, apparently move forward and backward in the Zodiac; conſequently, ſometimes they appear Stationary, and Mercury and Venus have their greateſt Elongations from the Sun, Eaſt and Weſt of him.

3. Mercury continues Retrograde during 18 Days, Venus 40 Days, Mars 3 Months, Jupiter 4 Months, Saturn $4\frac{1}{2}$ Months.

4. Mercury's greateſt Elongation or zodiacal Diſtance from the Sun is 28 degrees, that of Venus 48 degrees. At theſe times their Horizontal Parallaxes are nearly equal to that of the Sun.

XXI.

Of the Moon's Separation from the Sun, and Stars within the Zodiac; as ſeen on Land and at Sea.

1. The Moon's Separation from the Sun and Fixed Stars, ariſes from the motion of the Earth in its Orbit, the motion of the Moon in her Orbit, the Parallax occaſioned by the Diſtance of the Moon and a Situation on the Earth, and the Situation of the Sun or Fixed Stars in the Heavens.

2. The Separation in Right Aſcenſion depends on the Moon's Latitude and Longitude, and the Sun or Star's place in the Zodiac.

3. The Moon's mean Daily Receſſion from the Sun in Longitude is 12d. 12m. The Sun's Daily Receſſion from the Stars in Longitude is od. 59m. Therefore, the Moon's mean Daily Separation from the Stars, both in Longitude and Diſtance when both are near the Ecliptic, is 13d. 11m.

XXII.

Of the Moon's Phaſes, as ſeen on Land and at Sea.

1. That Half of the Moon which faceth the Sun is illuminated, the other Half is dark. Therefore,

2. When the Moon is between the Sun and Earth ſhe diſappears. This is the time of her Change or New Moon.

3. When ſhe is in the oppoſite part of her Orbit, and not in her Node, ſhe appears wholly illuminated. This is the time of Full Moon.

4. Between the Change, and Full, ſhe appears Half illuminated. Theſe are her Quarters.

5. From the Change to the Full, ſhe is within 180 Degrees Eaſt of the Sun. From the Full to the Change, ſhe is within 180 Degrees Weſt of the Sun.

6. A Line joining the Cuſps or Corners is the Line of the Cuſps. A Line biſecting the Line of the Cuſps at right Angles, goes directly to the Sun.

7. When the Moon is in her Node, this Line continued behind the Moon coincides with the Ecliptic. When the Moon hath North Latitude, it comes out North of the Zodiac. When the Moon hath South Latitude, it come out South of the Zodiac.

XXIII.

Of the Poſitions which the Moon's Cuſps have at different Times and Places; as ſeen on Land and at Sea.

1. When the Moon is near the Ecliptic, the Poſition of the Line of the Cuſps depends on the Poſition of the Ecliptic; therefore at ſuch times, if one is perpendicular to the Horizon, the other is parallel to it.

2. The Tropics are Leſſer Circles parallel to the Equator, one paſſing by the north and the other the ſouth Boundary of the Ecliptic. The Polar Circles are each of them Half the Diſtance of the Tropics from each Pole.

3. Like Circles being ſuppoſedly drawn on the Earth and Seas, they become the Bounds of the Torrid, Temperate and Frigid Zones.

4. One Half of the Ecliptic Line in the Heavens is above and the other Half below the Horizon always, excepting in the Polar Circles; but thoſe Halfs are continually changing both the Degrees and Signs to which they belong and their Poſitions to the Horizon.

5. To Obſervers in the Torrid Zone the Ecliptic is perpendicular to the Horizon once eaſtward and once weſtward in the Space of every 24 Hours. At ſuch times the Zodiacal Luminaries are nearly perpendicular to each other, their Diſtances from each other in the Zodiac become nearly their Differences of Altitudes when on the ſame ſide of the Zenith; but nearly the Sum of their Co-altitudes when on different ſides of the Zenith.

6. The Moon being near the Ecliptic; when the Cuſps are nearly horizontal, the Sun is then near the ſame Vertical Circle. The like for Stars near the Ecliptic.

7. The Planets being near the Ecliptic, its Poſition is ſhewn nearly by any two of them when they are viſible; and when more of them are viſible, its Direction may often be judged of for large Diſtances in the Heavens.

XXIV.

Phænomena of the Zodiacal Stars and Primary Planets; as seen on Land and at Sea.

1. The Zodiac is a Space of a certain number of Degrees in Breadth northward and southward of the Ecliptic Line; and the Fixed Stars within this Breadth are called Zodiacal Stars.

2. The Breadth of the Zodiac is formed chiefly from the Planets greatest Latitudes and their Horizontal Parallaxes; this makes the Breadth about 14 Degrees from the greatest Heliocentric Latitude of Mercury, but if the Breadth of the Earth's Shadow in Lunar Eclipses, and certain large Fixed Stars near the Bounds of the Zodiac be taken in, its Breadth will be more. The Ancients gave it a Breadth of Twelve Degrees, the Moderns have extended it to Twenty Degrees.

3. Some Fixed Stars without the Zodiac are used as the Zodiacal Stars, when the Moon's apparent Recession (from West to East in 24 Hours) is either directly toward or from them and the Sun.

4. The principal Zodiacal Stars within the Zodiac, are,

1st. Aldebaran, a red Star having many small Stars round it, and the Seven Stars to its northwest.

2d. Pollux, about 45 Degrees eastward from Aldebaran, having Castor a Star of the same Magnitude, northwestwardly from it.

3d. Regulus, a red Star near the Ecliptic, about 37 Degrees southeastward from Pollux. This is the largest and southermost of four Stars in a crooked line northward.

4th. Spica, a very white sparkling Star, almost alone, near the Ecliptic, and about 54 Degrees southeast from Regulus.

5th, Antares, a large red Star, a little southward of the Ecliptic, about 46 Degrees southeast from Spica, having a line of small Stars southeastward like a Swan's Neck.

6th. Alpha and *Beta* in Capricorn, are two small Stars nearly north and south about 58 Degrees east of Antares. *Beta* as much north of the Ecliptic as Antares is south.

5. The Stars without the Zodiac used as Zodiacal Stars, are

7th. Alpha in Pegasus.

8th, Alpha in Aries; this was formerly near the beginning of Longitude, but it is now forward a whole Sign, by the Recession of the Equinoctial Points.

9th, Alpha in Aquila, a white sparkling Star about 25 Degrees north from the two in Capricorn, having two small Stars, northwest and southeast of it.

10th. Fomalhaut, a large Star about 45 Degrees south from Alpha in Pegasus.

6. The Mean Separation of the Moon from west to east of these Stars, is

in a Day,	13d.	11m.	0s.
an Hour,	0d.	32m.	58s.
a Minute,	0d.	0m.	33s.
2 Minutes,	0d.	1m.	6s.
4 Minutes,	0d.	2m.	12s.

7. *Mercury*, being near the Sun, is seldom seen. At his greatest Elongation, he appears like a large white Fixed Star.

8. *Venus*, near her greatest Elongation appears very large and refulgent, but near her superior Conjunction less splendid.

9. *Mars*, when farthest from the Earth, appears small, but when nearest to it large and red.

10. *Jupiter*, when nearest to the Earth appears large and refulgent, but when farthest from the Earth his splendor is lessened.

11. *Saturn*, when nearest the Earth, appears white like a large Fixed Star; farthest from the Earth the Magnitude appears diminished.

XXV.

Of Deceptions in the Theory of Gravity; arising from an imperfect knowledge of the Sun's true Figure.

1. When the most accurate Observations have been made of the Sun, Moon, Planets and Fixed Stars, and their places in the Heavens have been predicted from the most accurate allowances that can be perceived to arise through the Laws of the Solar System; it often happens that such predictions are not found correct, and it is difficult to assign the Causes whence the Errors have arisen.

2. In the Solar System, its Centre is at Rest; the Sun's Centre is sometimes but little less than a Semidiameter or 16 Minutes from that Centre, per Newton's Princ. Book 3. But the Primary Planets move in Ellipses which have their common Foci in the Centre of the Sun. Therefore,

3. In Figure 5. If *a* be the immoveable Centre of Gravity, *b* the Place of the Earth, and *c* a Fixed Star at an indefinite Distance; when the Centre of the Sun is carried to *d*, the Earth by respecting the Sun's Centre will be carried after it as to *e*; draw *fde* parallel to *cab*, so will *f* and *c* appear to coincide: but if the Earth be at *g*, whether the Sun's Centre be at *a* or *d* it is the same. Consequently, the Sun and Fixed Stars will have the same angular Distances whether the Sun's Centre be in or out of the common Centre of Gravity; and the Sun as a Planet accompanied with the Primary Planets, may move round the common Centre of Gravity in a Path and periodic Time, which is with the greatest difficulty (if ever) to be known.

4. By

4. By obſerving the two Diameters of the Sun under contrary directions, it has been found to have had about two Seconds Difference, when allowance has been made for Refraction; this makes the Sun's equatorial Diameter 934 Engliſh Miles longer than his Polar Axis.

5. By obſerving the Solar Spots, it appears that they are convex toward the Sun; near the Sun's Limb they move with ſuch an apparent Velocity as no way agrees with a Rotation of the Limb round the Sun's Axis; they apparently paſs by one another; theſe are from my own Obſervations. Hence no Rotation of the Sun round its Axis in 25d. 5h. can be inferred from them.

6. By ſuch an Oblate Sun, its Centre of Gravity will become changed under different Poſitions to the Orbits of Mercury, Venus and the Earth. The Sun's Force in cauſing the Receſſion of the Equinoctial Points produceth more than a fourth part of the whole, and what this may produce is not eaſily to be determined.

XXVI.

Of Refraction in Altitude; and the Cauſes whereby it becomes different through Changes in the Atmoſphere.

1. When Particles of Light come from the Sun, Moon, a Candle, Fire, or any other luminous Subſtance emitting them, they either go on in right lines, or are interrupted by ſtriking againſt other Subſtances, or are reflected, or refracted.

2. When the Subſtance againſt which they ſtrike is ſuch that it ſtagnates or abſorbs them, they produce no Senſation of Viſion; but when they are either reflected or refracted, and fall on the Eye of an Obſerver, they paſs to the Bottom of it, and excite in the Obſerver a Senſation of the Forms which the Subſtances have from which the Particles laſt came.

3. As the Poſitions of viſible Objects are indefinite, and the quickeſt Tranſition of the Eye from one Object to another, cannot diſcover any Defect in the Operations of theſe Particles, they are moſt wonderfully to be thought of; for, their Velocities muſt be beyond all human Conception, their Number every where too great, and their Magnitudes too ſmall, to be ever comprehended by Man.

4. When thoſe Particles paſs from one Medium to another, or through Mediums varying in Denſity, their rectilinear Direction is altered, by their being ſolicited or inclined to the denſer Parts, and with additional Velocity, from the Inſtant of their being within thoſe Limits.

5. The Atmoſphere ſurrounding the Earth and Sea is a Fluid of vaſt Rarity and Elaſticity. Its Weight compared with Water in equal Bulks, has been found from between 1 to 600 and 1 to 900, at different times and places. Its lower Parts muſt be more denſe than the upper, and its principal Parts reach to the Height of 50 miles.

6. Conſequently, Particles of Light coming from the Sun and paſſing through the Atmoſphere, in any other than a perpendicular Direction, do paſs through a Curve which is concave toward the Earth, and the Effect is greateſt when it comes horizontally, and nothing when it comes by the Zenith from either the Sun, Moon or Stars.

7. From theſe Cauſes, Refraction at any given Altitude is different at the ſame time at different Places, and variable at different times for the ſame place of either the Earth or Sea. It is greateſt in Winter, leaſt in Summer; greateſt in high Latitudes and leaſt in the Torrid Zone.

8. In a common Barometer adapted for the Temperate Zone, the Difference between the greateſt and leaſt Aſcent of the Mercury for Winter and Summer is three Inches nearly, and this anſwers to a change in the Horizontal Refraction, amounting to four Minutes of a Degree from Winter to Summer, by Sir Iſaac Newton. Yet, no inconſiderable part of this alteration in the Barometer ſometimes happens in a ſhort Interval of Time.

9. The Difference between the greateſt and leaſt Aſcent of the Mercury in a common Thermometer, for Summer and Winter is two Inches and half; and the like remarks may be made concerning its Alteration.

10. The Generation, Aſcent and Deſcent of Vapours prove Heterogeneity in the parts of the Atmoſphere; that, whilſt ſuch Inſtruments are taking their State below, they may be different from an homogeneous Diminution above, and thereby render the Concluſions in ſome meaſure erroneous.

11. But ſuch Errors although ſmall, may be almoſt annihilated by the following Conſiderations. In Figure 6. Let a be the place of a Spectator on the Earth, $bgfc$ an Arch of the Top of the Atmoſphere, d the higher and e the lower points of the Sun's Diameter, f the place where Particles of Light coming from d enter the Atmoſphere and g the place where they enter it from e. Then will d appear at h, and e at i, and the real Diameter de will appear at hi. Here, e is refracted more than d, and therefore, the Sun's Horizontal Diameter will meaſure about four Minutes more than his vertical one, near the Horizon.

12. A Table of Refraction in Altitude decreaſeth upward in a very irregular manner; it may be repreſented by a Curve not much unlike the Logarithmic Curve, with Perpendiculars for the Refractions, which ſhorten very faſt near the Horizon,

Horizon, after which it approaches nearly to a Right Line.

14. From these considerations, in two former Treatises I gave this Rule. Take the Horizontal Refraction at Sea, and when it agrees with that in a Table, the Refraction at any Altitude is shewn in the Table. When the Refraction taken is less than the Horizontal Refraction of the Table; find the observed Refraction in the Side of the Table, and it will appear how many Minutes you must go beyond any other given Altitude, to take out the Refraction required.

Example.

Obsd. Hor. Refrac.		24m.	30s.
Tab. Hor. Refrac.		34m.	30s.
Difference		10m.	0s.
Obsd. Alt.	8d.	10m.	
Corr. Alt.	8d.	20m.	
Refraction reqd.		6m.	15s.

XXVII.

Rules for calculating the Moon's Parallax in Altitude, and the Augmentation of her Horizontal Diameter, or Semidiameter; at any given Altitude.

1. The Moon's Parallax in Altitude depends on her Horizontal Parallax and her Altitude; the former is commonly defined to be the Angle which the Earth's Semidiameter subtends at the Moon, and the latter depends on the Elevation above the Horizon of the Earth or Sea.

2. In Figure 7. Let *a* be the Earth's Centre, *c* a place on its Surface, *afd* the true Horizon of *c*, *b* the Zenith, *bed* a Quadrant from the Zenith to the true Horizon, join *dc* and draw *dg* a Tangent to the Earth's Surface, and the other lines as per Scheme.

3. If *d* be the Moon, the Horizontal Parallax by the common Definition is the Angle *adc*, but in reality it is the Angle *adg* or *adh*, and not *adi*, although all these agree with the Definition.

4. But the Moon's Distance being about 60 of the Earth's Semidiameters, the Angle *cdg* or *hdi* will be very small, not a Second of a Degree, therefore it may be rejected as nothing.

5. If *ck* be the visible Horizon, the Angle *dae* is the true Altitude and *kce* the apparent Altitude; the Angle *bae* is the true Co-altitude, and the Angle *bce* the apparent Co-altitude; the Angle *ace* is Supplement to *bce*, and the Angle *aec* is the Difference of the Angles *bce* and *bae*, or of *dae* and *kce*.

6. As *ae* to Sine of Angle *ace*, so is *ac* to Sine of Angle *aec*. Therefore, As Radius to Cosine of the Moon's observed Altitude; so is the Horizontal Parallax to the Parallax in Altitude.

7. The Difference between the Moon's horizontal and zenith Diameters is very small, and the apparent Magnitudes of Bodies appearing under small Angles is nearly in the Inverse Ratio of their Distances, and these are nearly as the Sines of the Moon's Altitude. Therefore, As the Sine of the Moon's Altitude is to Radius; so is the Difference between her horizontal and zenith Diameters, to the Augment of her Diameter for that Altitude.

8. When either the Moon's horizontal Parallax, or her Diameter or Semidiameter are greatest, this Augment is greatest, when they are least the Augment is least; and hence they have their Mediums.

XXVIII.

Of Calculations in Nautical Astronomy; preparatory to the Discovery of the Longitude at Sea by them.

1. Calculations relative to Nautical Astronomy may be considered as of two kinds; first, such as are made for Predictions, and secondly, such as are necessary for immediate Application either on Land or Sea.

2. The Calculations for Predictions, require previous Observations and Tables of the most perfect kind, together with the Theory of Science and the best Methods of forming them. Such Materials are not always printed, but frequently they consist chiefly of Manuscripts, in the hands of particular Persons.

3. The Calculations for immediate Use, are chiefly made by the Logarithmic Tables; such have been published by me, in the shortest and best manner.

4. In the latter of these kinds of Calculations, it is necessary to have readily the Rules for Plane and Spherical Triangles, and the most ready way of working them, these are in my forementioned Treatise. In the Lunar Method of finding the Longitude at Sea, my Linear Tables will be found more easy and concise than any others that have appeared on the Subject.

XXIX.

Of the true angular Distance of Sun and Moon's Centres; and how it is found, by having the Cotemporary Observations.

1. The true Distance of Sun and Moon's Centres is the number of Degrees, Minutes and Seconds of a great Circle in the Heavens, that would appear to be intercepted between their Centres, could an Observation be made at the Centre of the Earth, and unaffected with Refraction and Parallax.

2. This true Distance will always become less on the account of Refraction only, but after that it

Refraction & Parallax.

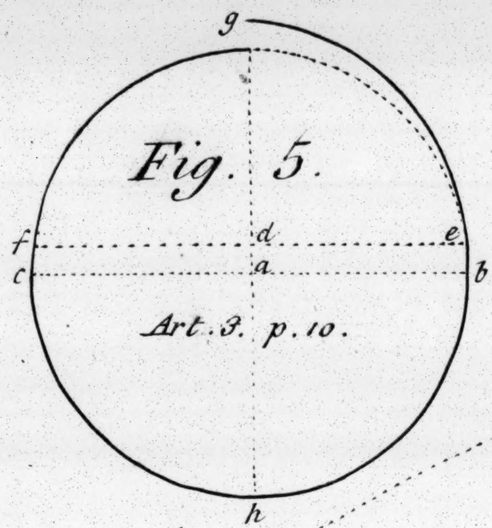

Fig. 5.

Art. 3. p. 10.

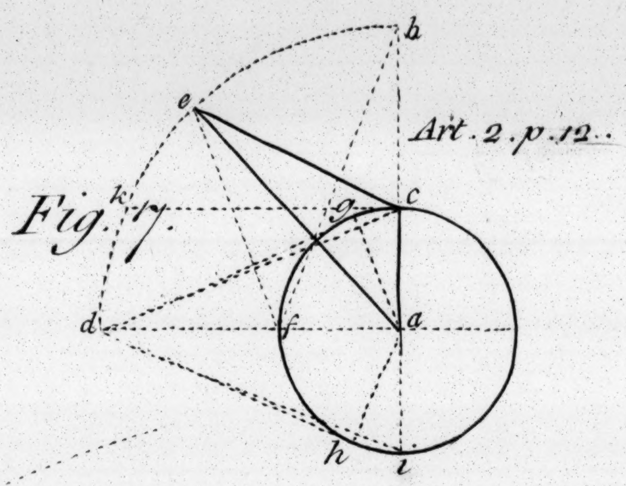

Fig. 7.

Art. 2. p. 12.

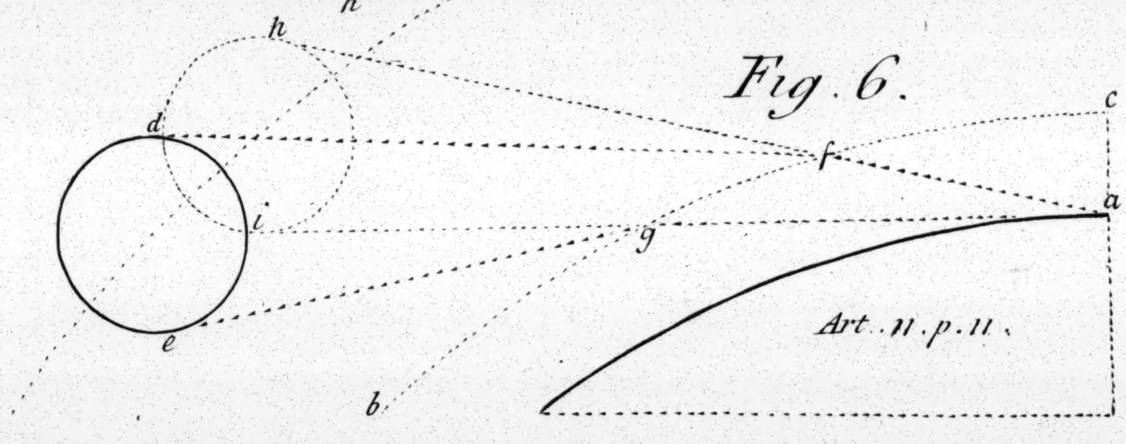

Fig. 6.

Art. 11. p. 11.

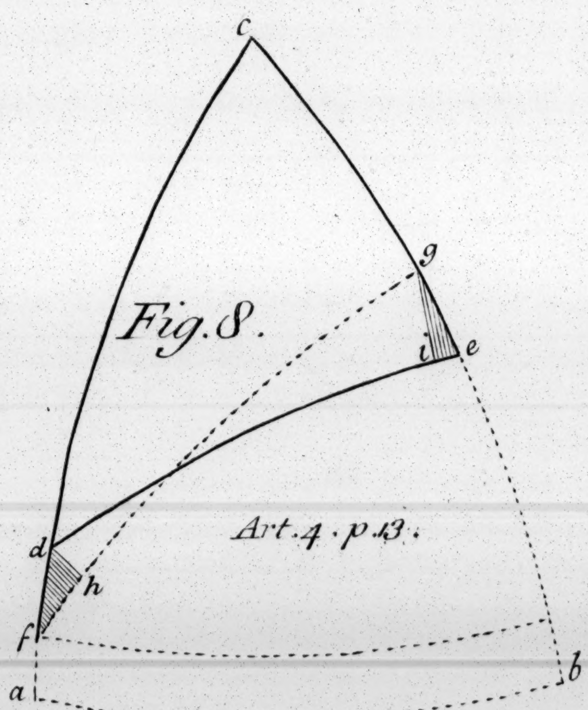

Fig. 8.

Art. 4. p. 13.

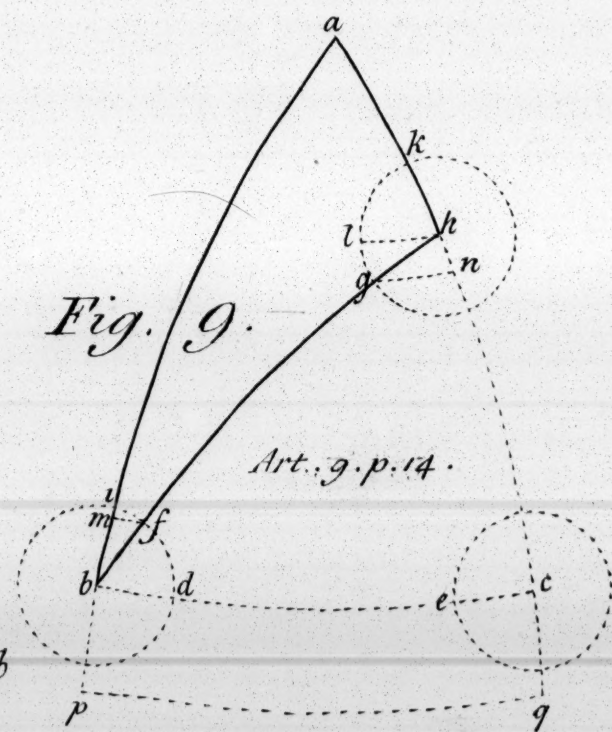

Fig. 9.

Art. 9. p. 14.

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it may be varied on account of Parallax, and therefore the whole Correction arises from the joint Effects of Refraction and Parallax.

3. In Figure 8. Let *ab* be an Arch of the Horizon, *c* the Zenith, *ad* the Altitude of the Sun's Centre cleared from Semidiameter and Dip of Horizon, *be* the Altitude of the Moon's Centre cleared from Semidiameter and Dip of Horizon; then, because Refraction apparently elevates, but Parallax depresseth the Sun and Moon, and because the Sun's Parallax in Altitude is always less than his Refraction in Altitude, let *df* be the Difference between the Sun's Refraction and his Parallax in Altitude. But, because the Moon's Parallax in Altitude is always greater than the Refraction in Altitude let *eg* be their Difference; so will *de* be the observed Distance of Centres and *fg* the true Distance cleared from Refraction and Parallax.

4. These Operations may be made different Ways.

First Method of finding the true Distance of Centres.

1. Having the Sides of the Spherical Triangle (Figure 8) *cde*, the Angle *c* may be found.

2d. Having the Difference of Sun's Refraction and Parallax in Altitude, the Side *cd* may be lengthened as much, this gives *cf*.

3d. Having the Difference of Moon's Refraction and Parallax in Altitude, the side *ce* may be shortened as much, this gives *cg*.

4th. Having the two sides *cf* and *cg* and the Angle *c*, the side *fg* may be found, which was required.

Second Method of finding the true Distance of Centres.

1st. Having the Sides of the Spherical Triangle (Figure 8) *cde*, the Angles *c* and *cde* may be found; and then, because the line *df* is very small, the Angle *cde* may (in many Cases without sensible Error) be called the Angle *cfg*.

2d. Find *df* and *eg* as before, and therefrom *cf* and *cg*.

3d. As Sine of Angle *cfg* (or *cde*) to Sine of Side *cg*; so is Sine of Angle *c*, to Sine of Side *fg* very nearly.

Third Method of finding the true Distance of Centres.

In Figure 8. Make *dh* perpendicular to *fg*, and *gi* perpendicular to *de*. Then,

1st. Get the Angles *c*, *ced*, and *cde* which put for *cfg*, as before.

2d. Get *cf* and *cg* as before.

3d. Consider the little Triangles *dfh* and *egi* as plane Triangles, and find in them the little Bases *fh* and *ei*; the first additive to and the second subductive from *de*, to give nearly the Distance *fg*, which was required. These Cases of adding or subducting the little Bases, vary according to the Angles at *e* and *d*, whether they are acute or obtuse.

5. Hence it appears, that, the Method of finding the Effects of Refraction and Parallax is easy and certain, requiring no other original Principles than those which Navigators either are or ought to be acquainted with (the Solution of Plane and Spherical Triangles) nor any other Logarithmic Tables than are or ought to be in every Ship at Sea; these are Circumstances which ingenious Persons will set some value on, seeing the Problem is not of the most compendious and easy kind, nor hath it been unobserved that there have been published by others various other Methods of solving it, which are not only difficult and tedious, but depend on Principles not agreed on amongst Mathematicians and Philosophers.

6. Nevertheless, Tables of an ingenious kind, adapted and calculated for solving this Problem in an easy and correct manner, cannot but be of singular Use to such as choose to decline entering into the Reasons or first Principles on which the Computations are founded. Indeed, such well constructed Tables, may at all times be readily applied, by all Persons, with more Certainty and Ease than a pure Analysis of the Problem admits of, through the Difficulties contained in it, and the want of Perfection, even in the readiest Computor.

7. Whenever such Tables are not at hand, or the manner of using them is not understood, there is no other Way of supplying their Use but by some original Method, amongst which three of the most capital ones have been before particularized. At such times and in such cases, the Calculator will do well to consider the Problem and its Parts as they depend on original Principles, and to apply such a Method of Solution as is most easy, best understood, and at the same time sufficiently correct for the purpose.

8. Under these Circumstances, although there be some additional Thoughts, the Labour will not be all lost, but in some Cases a nearer approach may be made toward the Truth; for, in solving the Question from its own Principles by an original way, there may be made such Allowances for Refraction, within a few Degrees of the Horizon, as the State of the Climate or Season requires, and thereby may be expunged the Errors which must necessarily arise from Tables confined to one Scale of Refraction, or a supposed constant State of the Atmosphere.

9. Whilst the Computor is bringing out the Solution by an original Way, he will depend much upon the Relations of the Sides and Angles of such Spherical Triangles as form the Parts to be

computed and lead to the Anſwer, and their Accuracy will depend much on the certainty of the Tables, which at leaſt ſhould give Arches and Angles to the neareſt Seconds of a Degree.

10. In this view, the firſt of the three foregoing ways, as ariſing from the whole Spherical Triangle, appears no way difficult, but as though it would be one of the moſt natural and eaſy Methods that can be purſued; for, it depends chiefly on the finding an Angle of a Spherical Triangle when the ſides are given, and then the finding a Side having two Sides and the contained Angle.

11. This is a natural and it would be the moſt correct Method that can be wiſhed for, if the Solution could ſuffer no Defect through the Relations of the Parts in the uſe of the Tables. But, ſhould the Angle at the Zenith happen to be ſmall and the Diſtance of the Centres great, an Error of a Second in the Zenith Angle may produce ten or even twenty times that Error in the Diſtance.

12. The Data that will produce ſuch (and it may be greater Errors) by ſuch a Method of Solution, frequently happen in the Torrid Zone, on account of the vertical Poſition of the Zodiac twice every Day; and therefore, however plain it may appear, or whatever Tables are at hand, this Method of Solution ſhould be practiſed with Caution.

XXX.

Of Corrections for the Data, in the Angular Diſtances of Sun and Moon.

1. When the Altitude of the Sun, the Altitude of the Moon and the Diſtance of Sun and Moon have been taken, due regard is to be had to the circumſtances attending them. The like when a Star is uſed inſtead of the Sun.

2. The Dip of Horizon is the number of Minutes and Seconds which the Viſible Horizon appears lower than it ſhould appear, by the Eye's Height above the Surface of the Sea; this is in its Table, and is always to be ſubtracted from the obſerved Altitude.

3. The Refraction in Minutes and Seconds is in its Table; this is always to be ſubtracted when uſed.

4. The Sun's Semidiameter is to be added to the obſerved Altitude of the lower Limb, but ſubtracted from the obſerved Altitude of the upper Limb; this is in its Table.

5. Some Writers on this Subject who have been ſcrupulouſly nice concerning the above corrections to Seconds and Parts of a Second, have omitted others; whilſt the Data retain at leaſt, the Errors ariſing from the Earth's Figure, and the Refraction for one part of Sun or Moon which belongs to another, and which frequently amount to many Seconds of a Degree.

6. A Spirit Level of a circular Form, when levelled has its Air Bubble in the Centre of the Level; then, the Fluid conforms to the Earth's Figure, but a Plane Glaſs covering the Fluid is not coincident with the Surface of that Fluid, nor does the Fluid itſelf bear the Surface it would on a Spherical Earth, except at the Equinoctial and the Poles. Here,

7. In a Direction obliquely toward the Equinoctial, the Curvature is greater than in the oppoſite Direction toward the Pole; conſequently, the Tangents to the Centre of the Fluid all round, are not in the Plane of the Glaſs, and Altitudes taken by both will differ, when taken under Oblique Directions.

8. When the Diſtance of Sun and Moon have been taken, and the Altitude of one of them amounts to but a few Degrees, it is proper to conſider how the other of them is ſituated, before the Refraction is taken out which is to be uſed for the Contact of the Limbs; for ſhould either Sun or Moon be near the Horizon the Refractions for the Contact of the Limbs muſt not be taken out as for the Centres, but a Correction made previous to the Solution, otherwiſe it may be inaccurate.

9. In Figure 9. Let a be the Zenith, b the Sun's Centre, c the Moon's Centre when their Altitudes are equal, then their neareſt Points of Contact will be d and e nearly, and the Effects of Refraction and Parallax will be nearly the ſame as for the Centres b and c. But, ſuppoſing b and h the Centres of the Luminaries, and f and g their neareſt Points of Contact, the Refraction and Parallax ſhould be for f and g and not b and h. Hence, when the Zenith Angle is ſmall, there may be ſome Difference between the true and ſuppoſed Refraction and Parallax in Altitude, through great Difference of Altitude.

10. To correct this Error. Draw mf, gn and lh parallal to the Horizon. By the two Co-altitudes and obſerved Diſtance, find the Angles at b and h, then is mb the Natural Coſine of the Angle abh or ibf to the Radius bf, which added to pb gives pm the Altitude of one Point of Contact; and hn is the Natural Coſine of the Supplement to ahb or of gha to the Radius gh, which ſubtracted from qh gives qn the Altitude of the other Point of Contact; for which reſpectively, the Refraction and Parallax in Altitude are to be taken. But, as this Correction may be thought tedious, I ſhall here only recommend in practice, to judge of the Angles by the Poſitions of the Luminaries, and make the Allowance accordingly, thus.

11. 1ſt. If the Luminaries have nearly equal Altitudes, the Refraction and Parallax in Altitude for their Centres, will be thoſe for their Points of Contact.

Contact. 2d. If they are on the same Side of the Zenith, those for the Contacts must be, one 16 Minutes higher and the other as much lower than for the Centres. 3d. If they are near the same Meridian on different Sides of the Zenith, those for the Contacts must be for 16 Minutes in each, higher than the Centres. 4th. If they are in an Oblique Position; first take a View of them, and then form a Judgment how many Minutes of a Degree they may have their Points of Contact higher or lower than their Centres, and allow accordingly. This, in usual Cases may be done without erring a Quarter of the Semidiameter, which is sufficiently exact.

XXXI.

Of Corrections for the Data; in Angular Distances of the Moon and a Star.

1. When the Altitude of a Star has been taken, it having no Semidiameter, the Refraction and Parallax in Altitude is for the Contacts, at the observed Altitude of the Star and the corrected Altitude of the Moon's Limb. In other Respects, the Corrections are to be made as for the Distances of Sun and Moon.

XXXII.

Of Cases, in finding the true Distance of Centres, of either Sun and Moon or Moon and Star.

See Figures 10. 11. 12. 13.

1. Because the Sun's Parallax in Altitude is always less than the Refraction at the same Altitude, as before noted, the Base of a little Right-angled Plane Triangle, will be additive to the observed to get the true Distance of Centres.

2. Because the Moon's Parallax in Altitude always exceeds the Refraction at that Altitude, the joint Effect of both always makes the Moon's true Altitude more than the observed. Consequently, 1st. When the Zenith Angle is acute and the Moon's Altitude less than the Sun's, the Base of the little Triangle is subductive. 2d. When the Zenith Angle is obtuse, and the Moon's Altitude greatest, that Base is subductive. 3d. When the Zenith Angle is acute and the Moon's Altitude is greatest, that Base is additive. 4th. When the Zenith Angle is obtuse and the Moon's Altitude is greatest, that Base is subductive.

3. A farther Correction may be introduced if (in Fig. 14) *ABC* be supposed the little Triangle itself, and *BF* be a perpendicular on its Hypothenuse; then, the Triangles *ABC* and *BFC* are similar, here, as *AC* is reduced to *AB* so is *AB* farther corrected by a smaller Quantity *CF*, and so may farther Corrections be introduced till they vanish.

XXXIII.

Of Methods for finding the True Distance of Centres, of either Sun and Moon or Moon and Star.

1. When the three cotemporary Observations have been taken, namely, the Altitude of the Sun or Altitude of a Star, the Moon's Altitude, and the Distance of Sun and Moon or of Star and Moon; in order to find the true Distance of Centres, there must be likewise given the Semidiameters and the Moon's Horizontal Parallax. Then, the Solution may be pursued either by the parts of the whole Triangle, or by finding such additional or subductive Corrections as will give the Truth.

2. If a Solution be pursued by the whole Triangle, the Degrees, Minutes and Seconds which enter into the Process, together with the Proportional Parts on which their nicer Subdivisions depend, will go on heavily, for they must be computed with the greatest Attention, otherwise, Error in adding or subtracting may be fatal. At last,

3. If the logarithmic Cosine of Half the true Distance is the Result; when the true Distance is small and a Table of Six places of Figures beside Index is used, there will be Error for Degrees of Distance, thus;

Deg. 27.	Error 4s.	or	2 Miles.
Deg. 18.	Error 6s.	or	3 Miles.
Deg. $13\frac{1}{2}$.	Error 8s.	or	4 Miles.
Deg. $9\frac{1}{2}$.	Error 12s.	or	6 Miles.
Deg. $5\frac{1}{2}$.	Error 20s.	or	10 Miles.

It may not be easy to compute on this Plan, less than six times for proportional Parts, and thereby introduce additional Errors, and the Relations of the Parts (as before observed) may make them more; and therefore, this Method, for short Distances, at least is not the most perfect; and how it may affect greater Distances is not easily to be determined.

4. If the Sun and Moon's or Star and Moon's Angles are found, there is but little danger of such or other Errors; for, those Angles may be readily found to a Minute of a Degree, and more accuracy is not wanted; then, the little plane Triangle and its Appendages, frequently exhaust all that is required. But,

5. When both these Angles are found, if the required Side is sought by Analogy of the Sines of the Angles and the Sines of the opposite Sides, there will frequently be Ambiguity in the Answer; for, it may be greater or less than a Quadrant, and the proportional Parts may make it more uncertain.

6. When, in the Solution of this Problem, a number of Terms arise to be disposed of by different Cases, their Variety will make the process difficult to the most able Calculator, and introduce

Error

Error through the number of particulars. It is ſtill worſe, when beſide theſe, Tranſitions are to be made from natural to artificial Numbers or the contrary, and when proportionals are to be found with the greateſt Care and Attention, whilſt the Tables to be uſed cannot give the Truth.

7. A ſimilar Remark may be made, on the application of Tables computed upon a ſuppoſed Equality of Refraction near the Horizon, for all times and places. In ſuch tables formed for whole Degrees, there will be at leaſt four Corrections generally occuring; namely, one for intermediate Minutes of Diſtance, two others for intermediate Minutes generally on both Altitudes of the Luminaries, and a fourth for the intermediate Seconds of the Moon's Horizontal Parallax. Whilſt theſe Corrections are making, ſome of them will be additive, others ſubductive, ſome great and others ſmall. All this tends to embarras the Computor, as he cannot with ſafety reject nor ſlightly paſs by any of them, it being very difficult for the moſt diſcerning Perſon, without Calculation, to judge what Error an Omiſſion of any of them may produce.

XXXIV.

Of Nautical Inſtruments, which either are or ought to be in a Ship at Sea.

The Principal Inſtruments which a Navigator at Sea ought to have, are as follow.

1. The Log-line; having its Knots 51 Feet diſtant from each other, and that ſuppoſed to contain ten equal Parts, although it be commonly otherwiſe divided.

2. The Sounding-line for meaſuring the Depth of the Sea, near the Coaſts.

3. Either a Half-minute Glaſs, Time-piece, or a Pendulum to meaſure that Interval, or what are better, a Spring Clock or Pocket Watch; applicable either for meaſuring by the Log-line, or in the making of aſtronomical Obſervations.

4. Two Steering Compaſſes; one in the uſual Form, the other ſo framed that its Card may be eaſily moved on the Needle that carries it, and made to ſhew the true Courſe without Variation.

5. The Azimuth Compaſs, for taking the Variation of the Compaſs, made in the beſt manner.

6. Hadley's Octant, for ordinary uſe in taking Altitudes; and the Sextant for nicer purpoſes in taking the Longitude.

7. The Nautical Ephemeris for the current Year; or any other Treatiſe ſupplying its principal uſe, when it cannot be had.

8. Charts of the Stars and of the Places where the Ship is expected to be, and Variation Charts of the Seas where it is expected to ſail.

9. A Spring Clock or Pocket Watch, to be kept as nearly to the Time at the Ship as Obſervations will permit; and which may be depended on for keeping one or two Hours of Time, with but ſmall Error. A Watch for keeping Equal Time during a long voyage, is not here mentioned, that belongs almoſt wholly to the Method of finding Longitude by a Time-keeper.

10. In uſing theſe Inſtruments; the Compaſs ſhewing the Points of the Horizon without Variation, will ſhew the Bearing of the Celeſtial Bodies near to and remote from the Meridian, nearly as they are to be underſtood; and thereby undeceive in making Obſervations at the moſt proper Times: and a Clock kept nearly to the Ship Time will anſwer the like good purpoſe.

XXXV.

Of Hadley's Sextant, its Perfections and Imperfections.

1. This Inſtrument as it is uſually made for Aſtronomical Obſervations at Sea, is not leſs than one Foot nor more than two Feet Radius; but, whether it be made of Wood or Braſs and intended for Uſe at Sea, its Radius ſhould not be leſs than fifteen Inches, in order to have its Subdiviſions eaſily underſtood.

2. When the Radius is not leſs than fifteen Inches and the Diviſions are finely cut in the moſt correct manner, by help of the Vernier or Nonius Diviſion, and the ſtricteſt Attention that can be employed, the Subdiviſions may be judged to near a Quarter of a Minute of a Degree. Such Diviſions are rarely to be met with, and therefore, to be able to judge to Half a Minute in a Foot Sextant, and to the third of a Minute in one of Eighteen Inches, is the beſt that can be expected.

3. On account of theſe Limits, ten Seconds of a Degree are not to be diſcerned on the Arch of the Inſtrument however accurately it is divided, and this alone will produce an Error of five Miles in Longitude at the Equinoctial; but if twenty Seconds be admitted for an Error in reading, the Error in Longitude will be ten Miles; and this may eaſily happen in reading the Subdiviſions when the Diſtance has been obſerved.

4. A Second Error to the ſame amount may ariſe in ſetting the Beginning of the Index to the Beginning of the Sextant, or in any other Mode of determining it; and, a third Error to as great an Amount may ariſe from an imperfect Junction of the Objects either between the quickſilvered and plain Part of the ſmall Mirror, or on the plain part itſelf; ſo here may be three Errors of one kind and of this Quantity.

5. The Amount of theſe Errors in their leaſt ſtate produceth an Error of fifteen Miles at the Equinoctial,

Cases of true Distance of Centres.

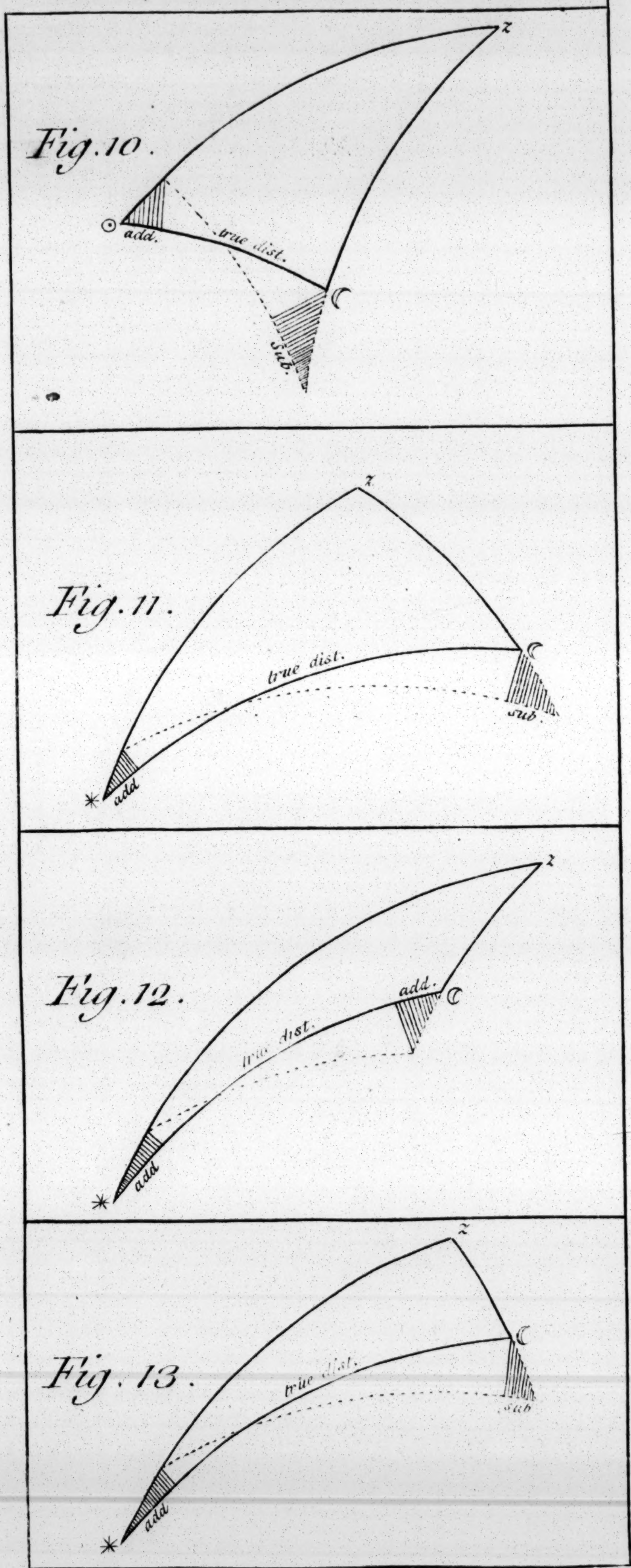
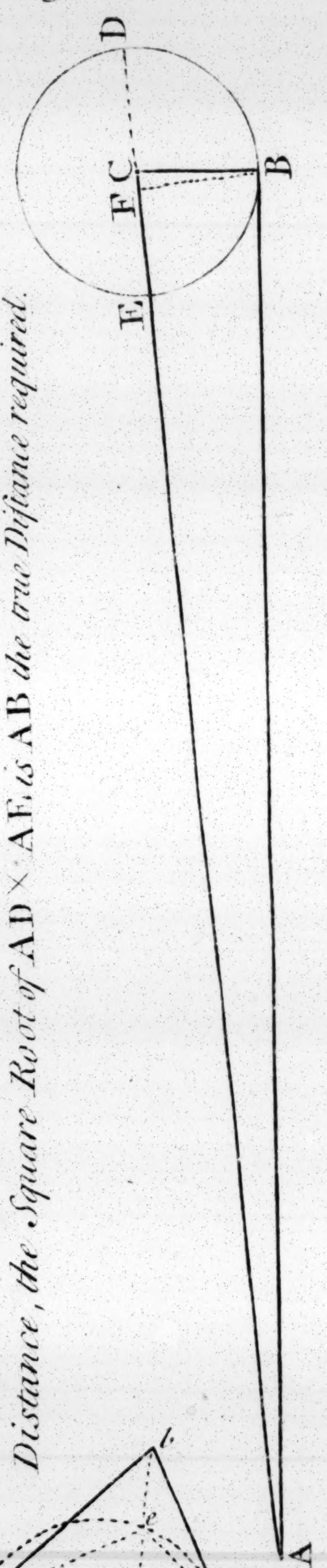
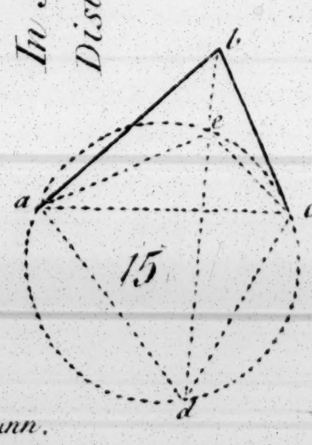


Fig. 14.



In short Distances, if B C be the little Perpendicular & A C the Distance, the Square Root of A D × A F is A B the true Distance required.



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Equinoctial, and may amount to Thirty Miles; nor is there any way of coming nearer the Truth, but by the Probability that may arise from a Medium. Farther,

6. If the Instrument be framed of such Materials and in such a manner, that it will not stand, but warp, shrink or expand in a Climate that is either extremely hot or cold; if there be any Error in the Divisions; any shake at the Centre, or the Parts there are not turned perfectly round; if the Glasses are not perfectly parallel Planes; if they happen to be bent or strained so as to alter their Planes; if the Eye is not at the place where the Adjustment requires it, or the Objects are not united at the proper place on the small Mirror; if the Limbs to be joined happen to be imperfectly defined, and other Causes of Error might be mentioned; then, many such of the same kind may arise in a single Observation, and it is very hard (if possible) to determine what Error may be in the Longitude deduced from them. Nevertheless, the Probability will be within the Limits before assigned.

XXXVI.

Of Accuracy in Observations at Sea, with Angular Instruments.

1. In the making of Astronomical Observations although the same Instruments are used by different Persons, it frequently happens that under like circumstances, Observers will differ; this may arise from various Causes. 1st. They may be short sighted. 2d, They may discern Objects best at a moderate Distance. 3d. They may see best at a remote Distance. Persons under either of these Cases may be assisted by Glasses, but the Effect may be different, and enable one or the other to perceive more distinctly, and thereby to form a Judgment concerning the Bounds or Limits of Objects and the greater or less number of Parts which their Magnitudes contain. Farther,

2. As the Eye of one Person may be better organized than that of another, so may the Faculties of the Mind, and this may be applicable not only to visible Objects, but to other Things, and amongst them to the smaller Quantities of Duration or Time. Both the Idea of Magnitude and Time, at least the Perceptions of them, are absolutely necessary in making Astronomical Observations, and a Deficiency in either of these, renders Observations imperfect although made with the best Instruments.

3. In the Royal Observatory at Greenwich 1769, I clearly saw the first external Contact of the Planet Venus with the Sun's Limb sooner, and the first internal Contact later than any of the other six Observers, by such an Interval, that the Time between the Contacts or passage of the Planet over the Sun's Limb, was three Quarters of a Minute of Time longer than a Medium of the other Observations.

4. When two fine Threads are stretched perpendicular to the Horizon, near the Plane of the Meridian, at the Distance of Eight Feet from each other, the Eye of an Observer being steadily fixed at a small Distance from the hindermost Thread in the Plane of both, may see the Instant when the Sun's Limb apparently toucheth the foremost Thread without erring two Seconds of Time in a single Observation. But,

5. When a Telescope is used of the same Length as the Distance of the Threads and magnifieth 80 times, the Contact cannot be taken without erring an Eighth of a Second of Time. Here, the apparent Swiftness is increased 80 times; and therefore, it might be taken to the 40th part of a Second of time, if there were no Inability in the Sight, Hearing, or both, to receive Impressions and report to the Mind, the adequate Ideas of swift Motion and Time. But, here both Sense and Judgment are limited, and four fifths of the whole is lost.

6. There is another Method of judging when either the Sun's Limb or a Star transits the telescopic Wire of a Transit Instrument in a fixed Observatory, by observing the Situation of the Limb or Star to the Wire both the Second of time before and after the Transit, and infering accordingly. Thus, if the Clock's Beat be when the Limb is short of the Wire by any apparent Space and at the next Beat it is past the Wire by double that Space, the Transit is for a third part of a Second of Time after the first Beat; but if it be doubtful whether the Limb is past half or a third of a Second of Time at the last Beat, the Medium will be but a little more than a tenth of a Second of Time different from the Truth.

7. This Method, depends on the Properties of Fractions which are nearly of the same Denominator, and is applicable to a great variety of Subjects. It is useful in reading the smaller Subdivisions of graduated Instruments, and the precise time when a swift Index passeth over the Division of a Dial Plate in a Time-keeper, and in some Cases, it may help to lessen the Errors of such Divisions; but will not correct any greater Errors that may originate in the Construction of Instruments.

8. The small Space which is apparently described by the Sun's Limb in a Second of Time, as viewed through the best Telescopes, and the quick return of the Clock's Beat, make it very difficult for the most acute Observer to determine

the Instant when the Limb or Star is on the Telescopic Wire, without erring a fifth of a Second of Time. Then, there will be three such Observations in assigning the Moon's observed Right Ascension, namely, one for the Star's passing the Wire, one for the Sun or another Star, and a third for the Moon; and these may produce an Error of three fifths of a Second of Time, which gives nine Seconds of the Moon's Recession, or an Error in Longitude at the Equinoctial of $4\frac{1}{2}$ Miles. There are five Wires in such Telescopes, at which, such Errors may arise, supposing them all placed right; and therefore, the time of Transit to the precise small part of a Second of Time cannot be determined.

9. If such Errors may arise from the most perfect state of Instruments and the Senses, what may be the Errors of Instruments imperfectly formed or adjusted, or Inability to make the best Use of them? 1st. In counting small portions of Time, the Mind with difficulty determines to the fifth part of a Second. 2d. The Eye with great difficulty discerns when the Sun's Limb or a Star, is within a Second of a Degree of the telescopic Wire. 3. The Clock may not have been exactly set to the Instant which was intended. 4th. It may err from its Rate of going in the Interval of Time. 5th. An Observer may err in judging of the Beats of the Clock, or Motion of an Index on a Dial Plate, as to the Instant they happen. Or, 6th. Of the Situations of the Limb or Star, near the Wire, at the Times they happen. 7th. The Central Wire of the Telescope may not move precisely in the Plane of the Meridian. 8th. The other Wires may not be correctly equidistant from the central one. 9th. The Mark for adjusting the Telescope may not be correctly in the Plane of the Meridian. 10th. There may be Error in adjusting the Horizontal Axis of the Transit Instrument, either by the Spirit Level or the Plumb-line. 11th. The Line of Collimation or Axis of Vision passing through the Telescope may either have been first out of the Plane of the Meridian, or it may have altered. 12th. The Telescope may become out of the Plane of the Meridian by its own Weight and Motion; otherwise, it would need no new Adjustment. 13th. There may be small Refractions parallel to the Horizon at particular times and places, by reason of Inequality in the Density of the Atmosphere. 14th. There may be a small Dilatation or Contraction of the Sun or Moon's Limbs according to the state of the Atmosphere. 15th. There may be a small Error arising for want of knowing correctly the Moon's Diameter, or even the Sun's; and another small Error may arise concerning the Stars being exactly on the Middle of the Telescopic Wire.

10. Of these fifteen Errors and others that might be mentioned, it is hard to determine which of them can be wholly annihilated by the exertion of the nicest Judgement and the sharpest Eye. It is certain that an Error of a Second of a Degree in the Moon's Daily Recession from the Sun and fixed Stars would be with the greatest difficulty (if at all) discerned, and that the Error may happen to be greater. But, were these all, and should the greater part of them happen too great or too little, they amount to fifteen Seconds of a Degree, and that is the Eighth Part of a Degree of Longitude at the Equinoctial.

XXXVII.

Of Corrections for Errors in observed Angular Distances of either Sun and Moon or Moon and Star, taken at the end of short and equal Intervals of Time.

1. The observed Angular Distance of Sun and Moon or Moon and Star, is commonly called a Sight. It is taken either with Hadley's Sextant or Octant and commonly expressed in Degrees and Minutes.

Of One or Two Sights.

2. When one Sight only is taken, it admits of no Correction, therefore in this Case it should be taken with all possible care and attention. When two Sights are taken their Medium is to be taken for the Middle-time between the two Observations, and this Medium admits of no Correction.

3. When three Sights are taken and the Medium of the first and third agrees with the Middle Sight, and the two Intervals of Time are equal, this Medium is for the Middle-time of Observation without any Probability of Error. But, when three Sights are taken with unequal Intervals; or, when the Intervals are equal and the Mean of the Extremes differs from the Middle Sight, there is reason for farther Correction.

4. During the Time allotted for taking the Distances of Sun and Moon or Moon and Star, they either increase or decrease in Distance, very nearly equal Spaces in equal Intervals of Time; therefore if those Intervals are increasing or decreasing, a Series of Observations corrected by these Principles will be very near the Truth.

Of Three Sights.

5. When three Sights are taken at the end of equal Intervals of Time; Add together the first and third Sight, half their Sum added to the Middle Sight, and that halfed Sum, gives the Truth nearly. Here, when the Medium of the extremes agrees nearly with the Middle Sight, it is very near the Truth. But, if Half the Sum of the Extremes, doth

doth not agree nearly with the Middle Sight; then, add it and the Middle Sight together, and Half their Sum will be the Truth nearly.

6. By this Method, the Correction may be made at once and the Half-sum will in most Cases come near the Truth for the Middle-time of the Observations. The Time may be taken at each Sight, by help of a common Watch, if the Time is required; but in the Lunar Method of finding the Longitude, no such Watch nor any Time-keeper is wanted, for two Assistants observing the Sun and Moon's Altitudes, or the Altitudes of Moon and Star, are sufficient. To young Practitioners, or others who would be exact and can observe expertly, a Watch will shew the supposed Intervals with Certainty.

Of Four Sights.

7. When four Observations are made at the ends of equal Intervals of Time, they may be considered as two Pair, each of which has its Medium; and therefore, the Half Sum of these two Mediums gives nearly the Truth. But, it is easier to add together the four Sights and divide the Sum by four. This Case admits of Examination; the Half Sum of any two, will nearly agree with the Sight between them, when they are taken near the Truth. Hence, a considerable Error in either of the four Sights may often be detected by Inspection. Then, the Medium of the whole, and its Substitution gives the Truth nearly.

Of Five Sights.

8. When five Sights are taken, at the ends of equal Intervals of Time, the Half Sum of the Extremes, the Half Sum of the second and fourth, and the Middle Sight, these should nearly agree; then, a third Part of their Sum is the Truth nearly. Or otherwise; take the Half Sum of the first and second, the Half Sum of the fourth and fifth, add these together and take their Half Sum, this should agree with the Middle Sight, if it doth not, add it and the Middle Sight together and Half their Sum is the Truth nearly.

Of Six Sights.

9. When Six Sights are taken at the ends of equal Intervals of Time, there will be four under the forementioned circumstances of that number; with two additional ones for the Extremes, which are to be treated as other Extremes.

Of Seven Sights.

10. When seven Sights are taken at equal Intervals, there will be two Extremes and five intermediate ones.

Of Eight, Nine, Ten, Sights.

11. When eight Sights, there will be two Extremes with six intermediates. When nine Sights, two Extremes with seven intermediates. When ten Sights, two Extremes with eight intermediates.

12. Consequently, when any Number of such Sights have been taken at the ends of equal Intervals of Time, by comparing a few of them together after the foregoing manner, it will readily appear what their Common Difference is nearly, and from this the most erroneous ones may be expunged, better ones put in their stead, and a nearer Approach made thereby to the Truth.

13. Hence, all the Sights may be added together and their Sum divided by the Number of Sights, and if the Quotients agree nearly with the Middle Term, it is probable there may be no great Errors in the Intermediates nor the Extremes; but for a farther Correction, the Sights nearest agreeing with this Medium may be retained, and others not agreeing left out, putting this Medium in their stead; and then their Sum divided by the Number of them, will be a nearer Approach to the Truth. Farther,

14. When an Error happens to be in either an Observation or Computation and it can be perceived, if it is more than the Truth it is commonly called *plus*, but if less than the Truth *minus*; and all errors do belong to one or the other of these two kinds, whether they are discernible or not. If such an Error is imperceptible, it can never be known, and the like may be said of a Combination of Errors partly correcting each other; but, if to such there be added a perceptible Error of the same kind whether plus or minus, the former imperceptible Error may thereby become doubled, trebled, or so augmented as to become visible. On the contrary, the addition of several perceptible Errors, may so correct each other, that the Truth may still remain undiscovered. Such Deceptions, may happen one time after another, and make the Truth itself remain undiscovered.

15. When two or more Errors happen, each of them may arise through Imperfection of the Senses, or the Mind, or Judgement, and depending on Physical Causes, which in their forming the minutest Ideas, may be different in different Persons; and therefore, in a Combination of Errors, if they have arisen from the best Observations that can be made, the Truth itself may not be discovered by a Medium, but a Quantity which in all probability differs from it by some small assignable Difference, but whether that be plus or minus, may ever remain uncertain.

XXXVIII.

XXXVIII.

Of Time, as kept by Clocks and Watches; either at Sea or on Land.

1. Siderial Time is meaſured by the apparent Revolutions of the Fixed Stars; this at all times expreſſeth equable Intervals of Duration.

2. Solar Time is meaſured by the apparent Returns of the Sun to the ſame Meridian; this, at all times is unequable, and the intermediate Subdiviſions partake of the Unequability.

3. Mean Solar Time is equable Time through the Year, beginning and ending with Solar Time.

4. The Pendulum Clock, is intended for ſhewing the Diviſions of Equable Time, which may be either Siderial Time, Mean Solar Time, or any other.

5. The Watch, whatever be its Size or whatever Expence attends its Conſtruction, in its moſt perfect ſtate, is intended for ſhewing the Equable Diviſions of Time; but, it is commonly adapted for Mean Solar Time.

6. The Earth's Diurnal Motion, and its Annual Motion round the Sun, meaſure the Periods of Time and their Subdiviſions as ſhewn by the Fixed Stars and the Sun. Theſe Motions have no other Inequalities than what ariſe from the Laws of Nature; which being known, there can be no doubt of their Truth and Certainty.

7. In a Pendulum Clock; if there be Inequality or Imperfection in forming the Wheels, any Flaw or Defect in the Teeth or Pinions; if the Oil is rancid with which they are touched, or what is worſe, become thick with Friction; if the Weight be not rightly proportioned; or if there be not the moſt perfect proviſion for the Effects of Heat and Cold on the Pendulum; if there be any Shake in the Pendulum, or if the Caſe be not fixed firmly as to a Block of Stone; if the Changes in the Atmoſphere cauſe any Alteration in the Motion of the Pendulum; if the Dial Plate be not accurately divided, or there be any Shake in the Indexes; from any of theſe Cauſes and others that might be named, Errors may ariſe, and it is not eaſy to determine to what they may amount.

8. In the Watch, Errors may ariſe from many of the forementioned circumſtances and beſides from the following Cauſes. If there be any Inequality of Denſity, Elaſticity or Coheſion in the Parts of the Main or the Pendulum Springs, any Flaw whereby they become apt to ſnap, bend or yield unexpectedly; if the Teeth or Pinions are imperfect, or if Duſt, moiſt or corroſive Air gets between them; if the Pivots or their Holes be clogged; if the Kirb of the Pendulum Spring be imperfect, or the Motion varies under different Poſitions; if there happens a Shock, Stroke or Fall, a Strain in winding up, or it be forgot: after ſuch Accidents and others that may happen, the Machine may not only ſhew erroneous Time; but, when it ſtops, all Time kept thereby is loſt and irretrievable.

9. The Motions of the Celeſtial Bodies are the adequate Meaſures of Time, and every Inſtant, their Poſitions are adequate Expreſſions for the preſent Inſtant of Time. Their Poſitions can be taken frequently, and the preſent Inſtant of Time truly found by them, whether it reſpects the Place of the Obſerver or other Places on the Earth or Sea.

XXXIX.

Of the Moon's Place in the Heavens, as taken at Sea.

1. The Moon's Place in the Heavens is her Situation in the Zodiac both in Longitude and Latitude at any Inſtant of Time. At the ſame Inſtant ſhe has a certain Diſtance in the Arch of a Great Circle of the Heavens from the neareſt Zodiacal Stars, but is continually receding from ſome and approaching toward others.

2. The Angular Diſtance of Sun and Moon in the Day and of Moon and Zodiacal Stars in the Night, is taken by Hadley's Sextant; this is performed by bringing the Image of the Sun's neareſt Limb to the neareſt Limb of the Moon, in the Day Obſervation; and that enlightened Limb of the Moon, to the Zodiacal Star in the Night Obſervation.

3. In taking either of theſe Diſtances, the utmoſt care and attention ſhould be employed, and in order to be the more exact, they ſhould be taken as nearly at equal Intervals of Time as poſſible. That Interval may be from one to two Minutes of Time, or leſs or more; but, if Clouds or any other Cauſe ſhould obſtruct this Regularity, the additional Interval ſhould be noted as twice, thrice, or more or leſs than the common Interval, and conſidered accordingly.

4. When any Number of ſuch Sights have been taken, they ſhould be carefully corrected by the Rules which have been given, and the Medium of the whole will be the obſerved Diſtance for the Mean Time or Interval of the Obſervations. If the Obſervations are divided into two or more Parts, the Mediums of each will be for the Mediums of the correſpondents Intervals of Time.

5. By the obſerved Diſtance of the Limbs, proceed to find the true Diſtance of Centres, and compare it with any correct Prediction; then, the Prediction gives the Solar Time for its Meridian when ſuch true Diſtance would happen, and that is hereby known at the Place of Obſervation, although the two Places are far remote from each other.

6. A good Obſerver with a good Sextant, may take and write down ſeparately, five Diſtances of Sun and Moon's Limbs in five Minutes of Time; but, if five Diſtances be taken in ten Minutes of Time, it will not be too long. When the Intervals are longer, it will be beſt to compute, either ſeparately or for ſmall intermediate Intervals, becauſe the Altitudes correſponding with them will give Reſults nearer the Truth.

XL.

The principal Definitions and Theorems of Plane Trigonometry; as printed in the Author's Treatiſe on Logarithms.

1. In a Right-angled Plane Triangle, the longeſt Side is the Hypothenuſe, the ſhorteſt Side the Perpendicular, the other Side the Baſe.

2. In any other Plane Triangle, the longeſt Side is the Baſe, the other Sides are the two Legs. The Perpendicular falls from the greateſt Angle perpendicularly on the Baſe, dividing it into its greater and leſſer Segments. The Altern Baſe is the Difference of the Segments.

3. In a Right-angled Plane Triangle; the Hypothenuſe, Baſe and Perpendicular, are to each other reſpectively, as the Radius, Sine, Co-ſine, Tangent or Secant reſpectively, which they repreſent when any one of the Angular Points is made the Centre of a Circle.

4. In all Plane Triangles, the Sides are in Direct Proportionality to each other, as the Sines of their oppoſite Angle.

5. In all Plane Triangles, one of the Angles being ſubtracted from One hundred and eighty Degrees, leaves the Sum of the other Angles, and half this is their Half Sum; conſequently,

6. The two leſſer Angles of a Right-angled Plane Triangle, are Complements to each other.

7. In all Plane Triangles; as Half the Sum of two Sides, is to Half their Difference, ſo is the Tangent of Half the Sum of their oppoſite Angles, to the Tangent of Half the Difference of thoſe Angles.

The Half Difference added to the Half Sum, gives the greater Angle, and ſubtracted from the Half Sum leaves the leſſer Angle.

8. In any Plane Triangle; as Half the Baſe is to Half the Sum of the Legs, ſo is Half the Difference of the Legs to Half the Altern Baſe.

Half the Altern Baſe added to Half the Baſe, gives the greater Segment; and ſubtracted gives the leſſer Segment.

XLI.

Definitions and Theorems of Spherical Trigonometry; as Printed in the Author's Treatiſe on Logarithms.

1. In a Right-angled Spherical Triangle; the longeſt Side is the Hypothenuſe, the ſhorteſt Side the Perpendicular, the other Side the Baſe.

2. In any other Spherical Triangle; the longeſt Side is the Baſe, the other Sides are the two Legs. The Perpendicular falls from the greateſt Angle perpendicularly on the Baſe, dividing it into its greater and leſſer Segments. The Altern Baſe is the Difference of the Segments.

3. In a Right-angled Spherical Triangle, the Right Angle is rejected as Nothing; then, the Complement of the Hypothenuſe, the Complement of the Angle at the Baſe, the Complement of the Angle at the Perpendicular, the Baſe, and the Perpendicular; theſe make the Five Circular Parts.

4. The Log. Sine of a Middle Part added to Index Ten, makes the Sum of the Log. Tangents of the two next adjoining Circular Parts; alſo, it makes the Sum of the Log. Co-ſines of the two Circular Parts which are not next adjoining.

5. In all Spherical Triangles; the Sines of the Sides are in Direct Proportionality to each other, as the Sines of their oppoſite Angles.

6. In all Spherical Triangles; as the Co-ſine of Half the Sum of two Sides, is to the Co-ſine of Half their Difference, ſo is the Co-tangent of Half the included Angle, to the Tangent of Half the Sum of the oppoſite Angles. And,

As the Tangent of Half the Sum of the two Sides is to the Tangent of Half their Difference; ſo is the Tangent of Half the Sum of the oppoſite Angles, to the Tangent of Half their Difference.

The Half Difference added to the Half Sum, gives the greater Angle oppoſite to the greater Side; and ſubtracted, leaves the leſſer Angle oppoſite to the leſſer Side.

7. In any Spherical Triangle; as the Tangent of Half the Baſe, is to the Tangent of Half the Sum of the Legs; ſo is the Tangent of Half the Difference of the Legs, to the Tangent of Half the Altern Baſe.

Half the Altern Baſe, added to Half the Baſe, gives the greater Segment; and ſubtracted, gives the leſſer Segment.

8. In any Spherical Triangle, whoſe three Sides are given to find an Angle;

Add together the Complements-arithmetical of the two Sides including the required Angle, the Log. Sine of Half the Sum of the three Sides, the Log. Sine of the Difference between the Half Sum and the Side oppoſite to the Angle ſought; Half the Sum of theſe four Logarithms is the Co-ſine of an Arch, which Arch being doubled is the Angle ſought.

9. In any Spherical Triangle, whoſe three Sides are given to find an Angle;

Take Half the Difference of the Sides including the Angle ſought, add it to Half the Side oppo-

ſite to the Angle ſought, and call it the Sum. Subtract this Half Difference from the Side oppoſite to the Angle ſought, and call it the Remainder.

Add together the Complements-arithmetical of the Sides including the Angle ſought, the Log. Sines of the Sum and Remainder; take Half the Sum of theſe four Logarithms, and it is the Log. Sine of an Arch, which Arch being doubled is the Angle ſought.

10. In any Spherical Triangle, whoſe Sides are given to find an Angle.

Take the Difference between each of the Sides including the Angle ſought, and the Half Sum of the three ſides, and call them the two Remainders.

Add together the Complements-arithmetical of the containing Sides, and the Log. Sines of the two Remainders, Half the Sum of theſe four Logarithms is the Log. Sine of an Arch, which Arch being doubled, is the Angle ſought.

XLII.

Of the apparent Diurnal Motions of the Celeſtial Bodies; in the Right, Oblique and Parallel Spheres.

1. Wherever a Perſon is on the Surface of the Earth or Sea, the Celeſtial Bodies appear either to move round him or the Earth's Axis produced, in a little more or leſs than a Day, and according to the Directions in which they appear to move during that Interval, the Sphere of the Celeſtial Rotation is ſaid to be either Right, Oblique, or Parallel.

2. When a Perſon is at the Earth's Equinoctial Line, the parts of the Celeſtial Equator, appear to aſcend perpendicularly to the Zenith, and to deſcend perpendicularly from the Zenith to the Horizon; directly from Eaſt toward the Weſt. This is called the Right Sphere. Hence,

3. In the Right Sphere, the Celeſtial Poles are in the Horizon, north and ſouth, the Zenith is in the Celeſtial Equator, and wherever a Celeſtial Body is in the Heavens, it becomes viſible at ſome time or other, during the Earth's diurnal Rotation, if the Horizon is unincumbered.

4. In the Right Sphere, the Celeſtial Bodies which have Declination, apparently deſcribe Semicircles above the Horizon which are leſs than the Celeſtial Equator; and a Spectator on the Earth or Sea, perceives their Semidiurnal Arcs as he would the Semicircular Baſe of a Cone, his Eye being at its Vertex.

5. The places on Land and at Sea, through which the Equinoctial Line paſſeth, are part of Africa, the Indian Ocean, the Iſlands Sumatra, Borneo and Celebes, the Pacific Ocean or Great South Sea, the north part of Peru and Amazonia in South America, the ſouth part of the Atlantic Ocean and the north part of the Ethiopic Ocean. The Lands in thoſe parts are inhabited chiefly by barbarous Nations, who have but little knowledge of Aſtronomy; the Oceans and Seas are frequently navigated by Europeans.

6. Seeing that by a Deception of the Sight, Objects near the Horizon ſeem to be more remote from each other than when they are near the Zenith and ſeen under equal angular Diſtances; it follows, that in the Right Sphere the Horizontal Diſtance of the Tropics appears to the Sight greater than the ſame Diſtance overhead from north to ſouth of the Zenith, and that thereby the ſemidiurnal Ares of the Celeſtial Bodies between the Tropics, do appear to Spectators there, nearer to a Verticality through this Deception than they otherwiſe would; this is an Advantage in the Practice of Nautical Aſtronomy.

7. In the Oblique Sphere, the Celeſtial Pole elevated above the Horizon, has a great variety of Poſitions, from the leaſt to the greateſt Elevation, both for the north and ſouth parts from the Equinoctial. The Lands and Seas where thoſe Elevations are, extend from near the Equinoctial Line on the Earth, to near the Earth's north and ſouth Poles, therefore they are numerous and of great extent.

8. In the Oblique Sphere, by Night, the Stars which are not more remote from the elevated Pole than the Latitude of the place, will become circumpolar Stars, and therefore they never riſe nor ſet, but are conſtantly above the Horizon. This ſuggeſts a very eaſy method of finding nearly the place of the Pole itſelf by them; for, if their Poſitions on a Chart of them be compared with their apparent Poſitions in the Heavens at any time, the place of the Pole itſelf may be nearly judged of.

9. In the Oblique Sphere, the ſemidiurnal Arcs of the Stars which Riſe and Set, will not be Semicircles as in the Right Sphere, except the Stars be in the Celeſtial Equator; but, the ſemidiurnal Arcs of Stars that are under the Horizon a ſmall time, will be almoſt whole Leſſer Circles of the Sphere; and the Arcs of Stars that are under the Horizon almoſt the whole day, will be ſmall parts of Leſſer Circles of the Sphere, but the Arcs of thoſe between the Celeſtial Equator and the elevated Pole, will be greater than Semicircles of Leſſer Circles in the Sphere; and the ſemidiurnal Arcs of thoſe between the Equator and the depreſſed Pole, will be leſs than Semicircles of Leſſer Circles in the Sphere.

10. In the Oblique Sphere, Deceptions ariſing from the various Poſitions and angular Diſtances of the Celeſtial Bodies will be very great, when ſome

ſome are near the Horizon and others toward the Zenith, on account of their oblique apparent Motions; theſe and other circumſtances tending to deceive the Judgment, ſhould be carefully attended to previous to the taking of their altitudes or angular Diſtances, in which all ſuch Deceptions vaniſh and amount to no Quantity.

11. Could a perſon be ſituated at either of the Earth's Poles, the Celeſtial Bodies would apparently move round him parallel to the Horizon; therefore, this is ſaid to be a Parallel Sphere.

12. Before an Obſerver can be duly ſituated for the Parallel Sphere, he muſt either travel or be navigated through places near the polar Regions, which may have been frozen ever ſince the Creation, and where no human Beings have hitherto choſen to inhabit. From ſuch a Situation, it would be very difficult to return to any known part of the World with certainty; as, at the Earth's north Pole every Point of the Compaſs is South, and at the Earth's ſouth Pole every Point of the Compaſs is North, and a ſmall Error in the Courſe might lead to places of Deſolation or Deſtruction. Beſides,

13. At either of the Earth's Poles, the ſlow apparent increaſe or decreaſe of the Sun and even of the Moon, in Altitude during every Day, would render almoſt all Obſervations of the Altitudes ineffectual for aſtronomical purpoſes, and Altitudes of both Primary Planets and Fixed Stars, would be leſs applicable; ſo that (without mentioning other circumſtances) thoſe places ſeem to have been the laſt on the Earth and Seas, deſigned to be viſited by Man.

XLIII.

Of the Five Zones, and the Poſitions which the Celeſtial Bodies appear to have in them, at different times of the Day and Night, throughout the Year.

1. The Zones are particular parts or portions of the Earth and Seas, from Pole to Pole; and have their names according to the Tropics and Polar Circles of the Heavens, when ſtraight Lines are drawn toward their Bounds and Extents, from the Earth's Centre. Hence,

2. The Torrid Zone, is that part of the Earth and Seas, contained between the two Tropics on the Earth's Globe; therefore, this Zone is Forty-ſix Degrees and Fifty-ſix Minutes in Breadth, and has the Equinoctial Line in its middle, paſſing round the Earth. To all parts of this Zone, the Sun is at ſometime or other in the Year, vertical; ſo likewiſe are the Moon and ſome of the Planets, beſides which, theſe ſometimes come more north or ſouth toward the elevated Pole, according to their reſpective Latitudes, when their Nodes happen to be near the Equinoctial Points Aries or Libra.

3. The North Frigid Zone is the Surface on Land or Waters contained within Twenty-three Degrees and twenty-eight Minutes of the Earth's north Pole; the South Frigid Zone, is that contained within the like diſtance from the Earth's ſouth Pole. To theſe parts, there are greater varieties in the Poſitions of the Celeſtial Bodies, than at the Poles.

4. The North Temperate Zone, is that part of the Earth and Seas which is contained between the Torrid Zone and the North Frigid Zone. The South Temperate Zone is that part of the Earth and Seas, which is between the Torrid Zone and the South Frigid Zone.

5. The Temperate Zones admit the greateſt varieties in the Poſitions of the Celeſtial Bodies, and near the Bounds of the Zodiac, they ſometimes apparently approach nearer to the elevated Pole than the Zenith itſelf. In theſe Zones, ſome Fixed Stars never riſe, others never ſet, others riſe and ſet; here the Sun is always apparently ſeparating from the Fixed Stars, daily from Weſt toward Eaſt according to the Poſition of the Ecliptic and the order of the Zodiacal Signs; the Moon is daily ſeparating with a much greater apparent daily difference, ſomewhat near the ſame Path; the Primary Planets are near the ſame Path in the Heavens, but with this difference, they are either Stationary, Direct, or Retrograde in their Motions, according to their places in their Orbits and their Poſitions to the Earth in its Orbit. Hence,

6. As the Temperate and Torrid Zones, are moſt inhabited and much navigated, they are parts of the Earth and Seas, where the Celeſtial Luminaries appear advantageouſly for obſerving them, and applying ſuch Obſervations toward the Improvement of Geography and Navigation.

XLIV.

Of Heliocentric, Geocentric and Apparent Places, in the Heavens; for the Sun, the Primary Planets and Fixed Stars.

1. The Angular Diſtance of the Earth's Centre from the Beginning of the Earth's Orbit, is the Earth's Heliocentric Longitude; ſuppoſing it to begin at the beginning of the Sign Libra, when the Sun's Centre (as ſeen from the Earth) is apparently at the beginning of the Sign Aries.

2. The Heliocentric Longitudes of Mercury, Venus, Mars, Jupiter and Saturn, are reckoned on the Earth's Orbit, like thoſe of the Earth; but, their Heliocentric Latitudes are their angular Diſtances perpendicularly northward or ſouthward from the Plane of the Earth's Orbit.

3. Could

3. Could an Obſerver be at the common Centre of Gravity in the Solar Syſtem, and perceive the reſpective periodic Revolutions round that Centre, all the Motions would appear irregular according to the Compoſitions of the Laws directing them. But,

4. Could an Obſerver be at the Surface of the Sun, ſuch Parallaxes would ariſe, as muſt naturally ariſe from the Sun's Semidiameter and the Diſtance of the Sun's Centre from the common Centre of Gravity.

5. Could an Obſerver be at the Earth's Centre, the Order and Regularity of the Appearance at the common Centre of Gravity would be loſt, and inſtead thereof a multitude of Poſitions ariſe amongſt the Celeſtial Bodies, not to be accounted for by any other than Reductions to the Heliocentric Revolutions, in their reſpective Periodic Times.

6. When an Obſerver is ſituated on or near the Surface of the Earth or Sea (where he muſt be) he applies Inſtruments which are frequently defective; he views through an Atmoſphere, the qualities of which are not always eaſily to be known; Parallaxes of various kinds, often belong to the problem which is to be ſolved; and many other circumſtances there are, which muſt be judged of, otherwiſe the Concluſion may be in ſome meaſure erroneous.

XLV.

Of the Motion of Light, and how it affects the apparent Places of the Celeſtial Bodies; and of the Motion of Sound.

1. The Motion of Light, is ſo ſwift, that it is impoſſible to eſtimate it for ſhort Diſtances; nevertheleſs, by applying mathematical Calculations and aſtronomical Obſervations, the rapid Velocity with which Light moves through the Heavens may be eſtimated.

2. The Mean Motion of Sound in Time is
in one Second, 400 yards.
in two Seconds, 800 yards.
in three Seconds, 1200 yards.
in four Seconds, 1600 yards.
in four Seconds and half, a Mile.

3. The Motion of Light through the Heavens, is Seven hundred and ſeventy thouſand times ſwifter than the Motion of Sound. Therefore, the Motion of Light is ſaid to be inſtantaneous, for all Diſtances on the Earth and Seas.

4. If the Motion of Light was indefinitely greater than it is, or inſtantaneous, it would then come from the Sun and Fixed Stars, inſtantaneouſly without any Interval; but otherwiſe, whilſt the Earth is in Motion, and changing its Poſition to the Sun and Stars, theſe latter appear as though their Places in the Heavens, became thereby a little altered; this is called the Effect of the Aberration of Light; and never ſhews the Celeſtial Body more than a third part of a Minute of a Degree diſtant from its true place in the Heavens.

5. Beſides the Receſſion of the Equinoctial Points, the Earth's Axis has another ſmall wambling Motion, called its Nutation; whereby it never ſhews the Celeſtial Bodies more than Nine Seconds of a Degree diſtant from their true Places.

6. Both the Aberration and Nutation, are neceſſary to be allowed for, in determining the correct places of the Fixed Stars, to the greateſt Accuracy for any time of the Year; otherwiſe, the moſt accurate Inſtruments and Obſervations may ſhew thoſe ſmall changes in their Places.

XLVI.

Of Triangles on the Earth and Seas, and in the Heavens; applicable in the Diſcovery of the Longitude at Sea.

1. The Angle of a Spherical Triangle near the angular Point, may be conſidered as a Plane Triangle and treated as ſuch, when the Sides are very ſmall, becauſe ſuch little Triangles are formed on a ſmall ſpherical Surface which differs but little from a plane Surface.

2. When two Sides of a Spherical Triangle are very long, and the third Side is very ſhort; the whole Triangle will be formed on a Spherical Surface, which differs but little from a Plane Surface; and therefore, both the Sides and Angles of ſuch a Triangle may be treated as thoſe of a Plane Triangle, often without ſenſible Error.

3. Such Triangles are numerous on the Earth and Seas, on the Concavity of the Heavens, and amongſt the Heliocentric, Geocentric and Apparent Poſitions of the Sun, Primary Planets, Moon and Fixed Stars.

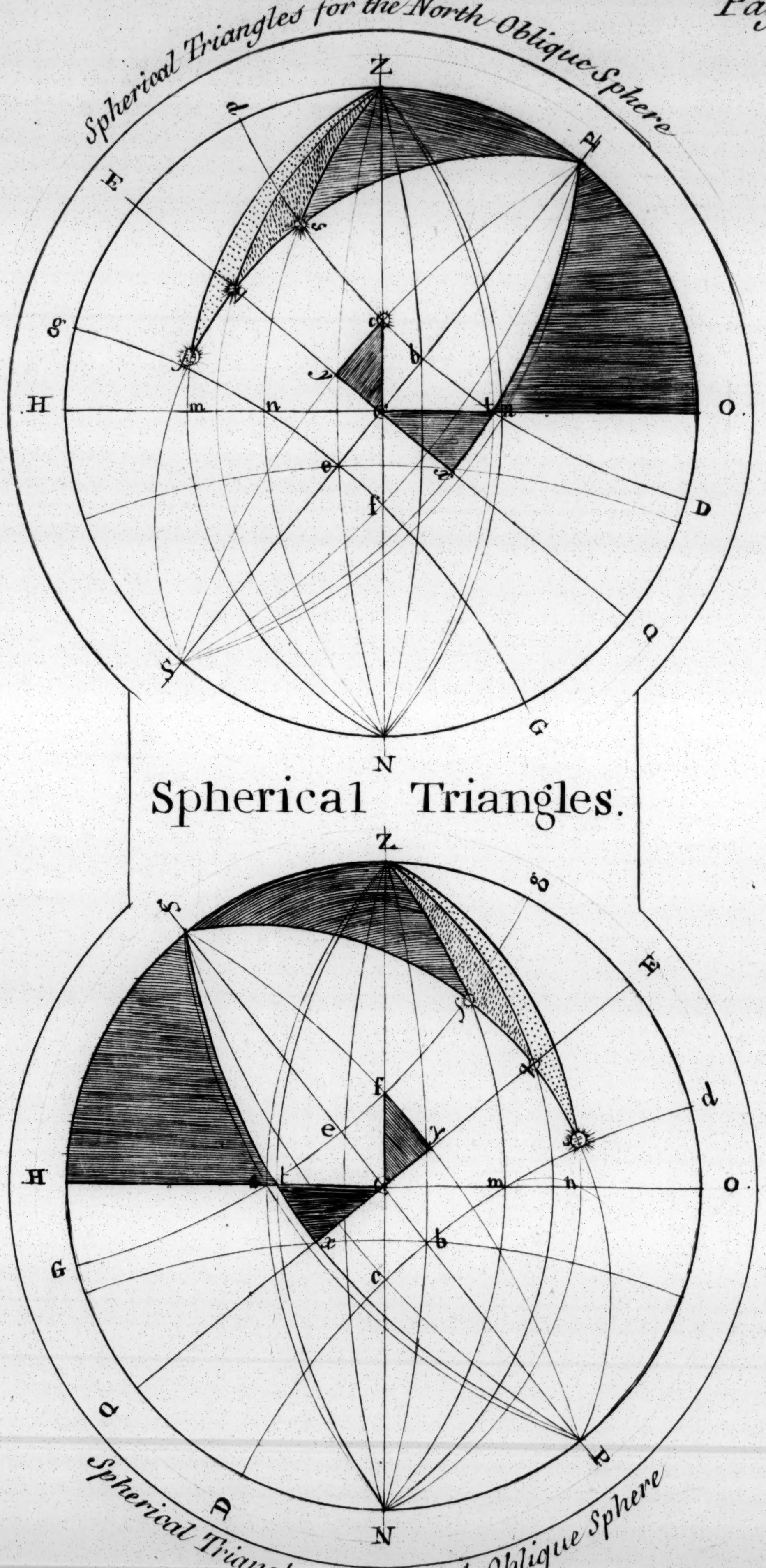
4. In Practical Navigation ſuch Angles and Sides form the Courſes and Diſtances ſailed. In Aſtronomy they form the Angles and Sides of ſmall Triangles ariſing from Refraction and Parallax. In the Lunar Method of finding the Longitude at Sea, they ſhew the Differences between Refraction and Parallax at the Sun and Moon's Limbs, and what it is at the Centres.

XLVII.

Of Spherical and Plane Triangles, formed on the Surface of the Earth and Seas, in North and South Latitude.

1. When two Places are both of them in north or ſouth Latitude, the two Co-latitudes and their Diſtance reſpectively, will be the three Sides of a Spherical Triangle. The Angle oppoſite to their Diſtance

Plate 5.
Page 24.



Spherical Triangles.

PZ s and PZ σ the general Triangle for Time and Azimuths.

Pot and Pxt the general Triangle for Amplitudes at Rising and Setting.

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Diſtance will be their Difference of Longitude, and each of the other Angles will be the Bearing of the other Place, with reſpect to the point under the elevated Pole, whether it be north or ſouth Latitude.

2. When two Places are one of them in north and the other in ſouth Latitude, the Latitude of one of them added to Ninety Degrees gives one Side, and the Latitude of the other taken from Ninety Degrees gives another Side, and their Diſtance will be the third Side of a Spherical Triangle. The Angle oppoſite to the Diſtance will be the Difference of Longitude; and the two other Angles, will be the reſpective Bearings of the two Places, with the point under the elevated Pole.

3. When three Places are ſituated on the Surface of the Earth or Sea, and the Arch of (nearly) a great Circle expreſſeth the Diſtance of any two of them, the Bearing of any two of them with reſpect to the third, will be expreſſed by a Spheric Angle, the angular Point being at the third Place; and the Differences of their Longitudes will be formed by Angles at the Poles, oppoſite to the reſpective Sides. Thus for any number of Places, however ſituated out of the Equinoctial and from the Poles.

4. When ſuch Diſtances become ſmall, the part of the Earth or Seas on which the Triangles are formed, differ but little from a Plane Surface, and conſequently the Angles then become inſenſibly different from Plane Angles.

5. In Figure 15. Let *a*, *b*, *c*, be three Objects on Land near a Sea Coaſt, *d* the place of a Ship at Sea in Sight of all three of thoſe Objects; *a d b* and *b d c* two angles taken with Hadley's Octant, Sextant, or any other Inſtrument, and ſuppoſe the Obſerver at *d* provided with a Chart of the Coaſt in which the three Places are ſhewn according to their true Diſtances from each other. Then,

6. Join *a c*; make the Angle *a c e* equal to the angle *a d b*; make the angle *c a e* equal to the angle *c d b*; and by the point of meeting *e* join *a e* and *c e*. Bring the three Points *a*, *e*, *c* into one Circumference, and through the points *b* and *e* produce a Right Line to that Circumference at *d*; ſo is *d* the place of the ſhip, in its due poſition to *a*, *b*, *c*.

7. If the three Objects are in a ſtraight Line, the ſame method of Solution may be applied; and the like if the Coaſt happens to be convex inſtead of concave toward the Sea; therefore, this method may be readily applied in many Caſes with Eaſe and certainty.

XLVIII.

Of the correct Altitudes of Sun and Moon's Points of Contact, in taking the Sun's Altitude, the Moon's Altitude, and angular Diſtance of Sun and Moon's Limbs.

1. It has been ſhewn in Section 30. how to correct this Error for general uſe, by judging nearly of the additional or ſubductive Minutes of Altitude from the apparent Poſition of the Point of Contact; but, when greater Accuracy is neceſſary the Truth itſelf may be had by the following Rule. See Figure 9.

2. 1*ſt*. To the obſerved Altitude of the Sun's lower Limb add his Semidiameter (after having ſubtracted the Dip of Horizon,) the Sum is the obſerved Altitude of the Sun's Centre.

2*d*. To the obſerved Altitude of the Moon's lower Limb add the Semidiameter, or if the Altitude of the upper Limb is taken, ſubtract that Semidiameter (after having ſubtracted the Dip of Horizon) this gives the obſerved Altitude of the Moon's Centre.

3*d*. To the obſerved Diſtance of the Limbs, add the Sun and Moon's Semidiameters, the Sum is the obſerved Diſtance of their Centres. 4*th*. Having the Co-altitude of the Sun's Centre, the Co-altitude of the Moon's Centre, and obſerved Diſtance of the Centres, get the Spherical Angle at the Sun's Centre and the Spherical Angle at the Moon's Centre, and conſider each of them as Plane Angles. 5*th*. With theſe Plane Angles and the Sun and Moon's Semidiameters, proceed (as is directed in Section 30) to find the Points either above or below the Centres, for which the effects of Refraction and Parallax in Altitude are to be allowed, and compute them for thoſe Altitudes and the obſerved Diſtance of the Limbs, and they will be thereby cleared from thoſe Errors.

3. The reaſon why this Rule ſhould be applied, in order to get the true Effects of Refraction and Parallax, is evident from the Figure itſelf; but, as it may be thought too tedious for general uſe, the Practical Method before mentioned may be uſed in its ſtead. However, when particular Accuracy is required, it will be of ſervice, and at all times will ſhew the Errors of Refraction and Parallax, computed without reducing them to the points of Contact in the Sun and Moon's Limbs, for which they are taken.

XLIX.

Of Corrections for the Deviation of Gravity, from a Direction toward the Centre of the Earth.

1. At any Place between the Equinoctial and the Poles, a Right Line drawn as a Tangent to the Earth and Sea, north and ſouth, will have its Perpendicular meet the Equinoctal Diameter between the Equinoctial and the Earth's Centre; and being continued upward, will have its Perpendicular directed ſo much nearer toward the Celeſtial Equator as the Deviation from a Direction toward the Earth's Centre.

2. At any Place between the Equinoctial and the Poles, a Quadrant of a Great Circle drawn

due Eastward and another drawn due Westward, will, at equal Distances from the given Place, have equal Curvature on the Surface of the Earth and Sea.

3. Near the Middle of the Quadrant, between the Equinoctial and either of the Earth's Poles, the Diviation will be greatest, and at the same distances from the Equinoctial as from either of the Earth's Poles the Deviation will be nearly the same number of Minutes and Seconds of a Degree.

4. This Deviation for every Degree of Latitude on the Earth and Sea, is nearly as follows.

Latitudes.	Deviation.	Latitudes.	Deviation.
D. D.	M. S.	D. D.	M. S.
0. 90	0. 0	23. 67.	13. 59.
1. 89	0. 41	24. 66.	14. 26.
2. 88	1. 22	25. 65.	14. 52.
3. 87	2. 2	26. 64.	15. 18.
4. 86	2. 43	27. 63.	15. 43.
5. 85	3. 23	28. 62.	16. 7.
6. 84	4. 3	29. 61.	16. 29.
7. 83	4. 42	30. 60.	16. 49.
8. 82	5. 22	31. 59.	17. 8.
9. 81	6. 1	32. 58.	17. 26.
10. 80	6. 39	33. 57.	17. 43.
11. 79	7. 17	34. 56.	17. 59.
12. 78	7. 55	35. 55.	18. 14.
13. 77	8. 32	36. 54.	18. 27.
14. 76	9. 8	37. 53.	18. 38.
15. 75	9. 44	38. 52.	18. 48.
16. 74	10. 19	39. 51.	18. 57.
17. 73	10. 53	40. 50.	19. 5.
18. 72	11. 26	41. 49.	19. 12.
19. 71	12. 58	42. 48.	19. 17.
20. 70	12. 30	43. 47.	19. 20.
21. 69	13. 1	44. 46.	19. 22.
22. 68	13. 31	45. 45.	19. 23.

5. The Error in Altitude arising from this Cause and its Reduction to the true Altitude as referred to the Earth's Centre, may be corrected by this Rule. As Radius, to the Sine of the Horizontal Distance of the Celestial Body from the true Meridian; so is the number of Seconds of a Degree of Deviation (in the above Table, for the Latitude of the Place) to the number of Seconds for Correction in Altitude.

L.

The Names of the principal Spherical Triangles which are formed in the Heavens, by the Apparent Diurnal Motion of the Earth.

1. Because the Right Sphere happens to none but such as are at the Equinoctial Line, and the Parallel Sphere to none but such as are at the Earth's Poles; the principal Phænomena of the Heavens, will be in the Oblique Spheres, north and south of the Equinoctial; and therefore, these will be the Phænomena of the principal parts of the Torrid and Temperate Zones, and of the habitable World.

2. In the visible Hemisphere of an Oblique Sphere, the principal Points of the Heavens are, the elevated Pole, the Zenith, and the Centres of the Sun, Moon, Primary Planets and Fixed Stars, or such of them as are above the Horizon. See Figure, Spherical Triangles.

3. In the Oblique Sphere, the principal Great Semicircles above the Horizon, are those of the Equator, Meridians, Vertical Circles, Ecliptic, and the whole Horizon.

4. In the Oblique Sphere, the Co-latitude, the Polar-distance, and the Co-altitude of either Sun, Moon, Planet or Fixed Star, form the three Sides of a Spherical Triangle; the three Angles of which are, the Hour-angle or Distance from the Meridian (opposite to the Co-altitude) the Azimuth-angle or horizontal Distance from the point under the elevated Pole (opposite to the Polar-distance) and the Angle at the Celestial Body (opposite to the Co-latitude). This commonly is an oblique Triangle.

5. When the Celestial Body is in the Equator, the Polar distance will be a Quadrant; when the Declination and Latitude are alike, the Polar-distance is less than a Quadrant, and when unlike it is more than a Quadrant. Hence, the Declination is subtracted from or added to a Quadrant, to get the Polar-distance.

6. When the Celestial Body is in the Horizon and the Latitude and Declination are alike, the Latitude, Polar distance, and horizontal distance from the Point under the elevated Pole, make three Sides of a Right-angled Spherical Triangle; and its Angles are, the Angle at the Celestial Body (opposite to the Latitude) a Right angle (opposite to the Polar-distance, and the Hour of Rising or Setting from Midnight (opposite to the Horizon). This Triangle is complemental to the Amplitude Triangle. For,

7. In the Amplitude Triangle, the Amplitude or Distance from the point of Rising or Setting to the East or West point of the Horizon, the Declination, and the Equatorial Distance from the East or West points of the Horizon, are three Sides of a Right-angled Spherical Triangle; and its Angles are, a Right-angle (opposite to the Amplitude) the Co-latitude (opposite to the Declination) and the Angle at the Celestial Body (opposite to the Equatorial Distance, or Ascensional Difference.)

8. In the East and West Azimuth Triangle, the Altitude in the Prime Vertical or East and West

Weſt Azimnth Circle, the Declination, and the Equatorial Diſtance from the Eaſt or Weſt point of the Horizon, are three Sides of a Right-angled Spherical Triangle; in which the Angles are, a Right-angle (oppoſite to the Altitude) the Latitude (oppoſite to the Declination) and the Angle at the Celeſtial Body (oppoſite to the Arch of the Equator.

9. In the Six of Clock (Hour-circle) Triangle, the Declination, the Altitude or Depreſſion, and Arch of the Horizon, are three Sides of a Right-angled Spherical Triangle; in which the Angles are, a Right-angle (oppoſite to the Declination) the Latitude (oppoſite to the Altitude or Depreſſion) and the Angle at the Celeſtial Body (oppoſite to the Horizon).

10. In theſe and other Spherical Triangles that are formed in the Oblique Spheres, the Horizontal Parallax of the Sun is ſo ſmall as to affect an Obſervation but little more than Half a Second of Time, and ſuch accuracy will be but ſeldom (if ever) wanted at Sea. The Horizontal Parallaxes of the Primary Planets will alſo be ſmall; and the Horizontal Parallaxes of the Fixed Stars will be nothing; therefore, all theſe may be applied as often as they are viſible, due regard being had to their Apparent daily Changes in Right Aſcenſion and Declination, Nevertheleſs,

11. The Altitudes of theſe and all others of the Celeſtial Bodies, whether taken on Land or at Sea, will be affected with the Deviation from a direction toward the Earth's Centre, according to their horizontal Diſtances from the Meridian of the place of Obſervation, and (although ſome additional Computation be neceſſary) ſhould be cleared therefrom, to come near the Truth.

12. The Moon's Horizontal Parallax, ſometimes exceeds a Degree, and therefore her Parallax in Altitude, and in angular Diſtances from the Sun, Primary Planets and Fixed Stars, is often very conſiderable; this muſt be as accurately allowed and computed for, as the nature of the Problem admits. The Analyſis of this Problem is in the former Sections, and its Solution may be had by the application of the Spherical Triangles which compoſe the Parts in their whole Extent, and of the Plane Triangles which form the ſmall Differences ariſing from Refraction Parallax and other Cauſes.

LI.

Of the Spherical Triangles, formed in the Heavens, by the apparent annual and periodic Revolutions of the Sun, the Primary Planets and Fixed Stars.

1. The principal Spherical Triangles ariſing from the annual and periodic Revolutions, are formed by Arches of the Celeſtial Equator, Arches of the Ecliptic, and Arches of the Celeſtial Meridians; their Angles being Arches of Great Circles at the diſtance of Ninety Degrees from the angular Points reſpectively.

2. The preſent Obliquity of the Ecliptic (or, what amounts to the ſame thing, the ſpheric Angle made by the Ecliptic and Equator at the two Points Aries and Libra) is very nearly, Twenty-three Degrees and twenty-eight Minutes; and, by comparing together all the moſt ancient and correct Obſervations that have been made (in my Treatiſe on Practical Aſtronomy) I have concluded that, this Angle is not become leſs than it was Two hundred and eighty Years ago, by more than a Minute of a Degree; and that therefore, there is no probability of its Alteration by more than that Quantity, for ſo many Years to come.

3. The Sun's geocentric or apparent Longitude in the Ecliptic, is therefore the principal thing to be predicted for any determinate time; this cannot be had but by the application of Tables made as conformable as poſſible with the moſt accurate Obſervations, and the Laws of Gravitation.

4. In the Spheric Triangle formed by an Arch of the Sun's Longitude, his Declination, and Right Aſcenſion; the Obliquity of the Ecliptic is oppoſite to the Declination, a Right-angle is oppoſite to the Longitude, and the Angle of the Sun's Poſition to the Equinoctial Point and the Equator, is the third Angle of a Right-angled Spherical Triangle. Here, the Longitude is reckoned from the neareſt Equinoctial Point, whether it be Aries or Libra.

5. As the foregoing Right-angled Spherical Triangles are formed, ſo are nearly ſuch Triangles formed by the Diſtances of the Primary Planets from their reſpective Nodes, their Latitudes either north or ſouth from the Ecliptic, and their Longitudes from their Nodes reſpectively. Here, their reſpective Parallaxes enter into the nicer Predictions.

6. The Fixed Stars being numerous and varioully ſituated, form Multitudes of both Right-angled and oblique-angled Spherical Triangles, with themſelves, the Celeſtial Poles, the Zenith, the Sun, the Primary Planets, the Moon, and various other parts of the Celeſtial Sphere.

7. The Moon's geocentric Place being always at no great Diſtance from the Ecliptic, and its Orbit being inclined to the Ecliptic; the Moon's diurnal Receſſion from weſt to eaſt, thereby becomes ſometimes toward Fixed Stars within and at other times without the Zodiac, in her ſwifteſt apparent diurnal Receſſion; therefore, theſe are occaſionally applied (when no Stars of the firſt Magnitude happen to be near the Moon's Path) in the Lunar Method of finding the Longitude at Sea.

LII. *The*

LII.

The End for which a Discovery of the Longitnde at Sea is designed and applied, in Practical Navigation.

1. The chief End and Design of a Discovery of the Longitude at Sea, is to enable Navigators to ſhorten long Voyages, or to ſail to and return from Ports and Places far remote from each other, whoſe Latitudes and Longitudes are known, in the ſhorteſt Time and ſafeſt manner poſſible.

2. In the Practice of this Problem after the moſt advantageous manner, it is not only neceſſary to determine the Latitude and Longitude of the place where the Ship happens to be, but to underſtand all other things of a ſcientific kind that may be of uſe for avoiding Dangers and bringing the Ship ſafe to her deſired Port.

3. Amongſt theſe, are the knowledge of particular Winds and other incidents, peculiar to the Oceans and ſeas to be croſſed; alſo, of the Coaſts, Iſlands, Rocks, Shoals, Sands, and other Obſtructions, in the Line of the Ship's track, and the beſt Methods of avoiding them, with the Courſes from one place to another which make the ſhorteſt Voyages; and knowing theſe particulars, to be careful in practiſing them under the greateſt Advantages, whatever Obſtructions are in the way, or the Accidents that may ariſe and lead into Dangers.

LIII.

To take the Latitude of a Place, either on Land or at Sea.

1. The Latitude of a Place is its Diſtance from the Equinoctial in the arch of a Meridian, either northward or ſouthward in Degrees, Minutes and Seconds. When the Place is north of the Equinoctial, it is in north Latitude; but when ſouth of the Equinoctial, in ſouth Latitude.

2. The Difference of Latitude of two places is the number of Degrees, Minutes and Seconds, which one place is more nothward or ſouthward than the other. If both places are in north or in ſouth Latitude, ſubtract one from the other, the Remainder is their Difference of Latitude. If one place is in north and the other in ſouth Latitude, add them together, and their Sum is their Difference of Latitude.

3. When a perſon looks exactly northward and takes the Altitude of a Celeſtial Body in Degrees and Minutes, it is its Altitude northward, and the Remainder to Ninety Degrees is the meridional Co-altitude or meridional Zenith Diſtance north. When a perſon looks exactly ſouthward, theſe Names are ſouthward.

4. When the Zenith Diſtance and Declination are both North or both South; their Difference is the Latitude and it is of the ſame Name with the Declination when the Declination is greater than the Zenith Diſtance; but otherwiſe, the Latitude is of a Name contrary to the Declination.

5. When the Zenith Diſtance and Declination are one North and the other South, their Sum is the Latitude, and it is of the ſame Name with the Declination.

6. In theſe Caſes, the Altitude is ſuppoſed to be taken when the Body is above the Pole, if it can have two Altitudes one above and another below the Pole. It is likewiſe ſuppoſed to have its Dip and Refraction ſubtracted, but its Semidiameter (and parallax in Altitude if it has any) added, after the Altitude is taken.

LIV.

To take Solar Time at a Place by an Altitude of the Sun.

1. Take two or three Altitudes of the Sun's lower Limb, and if they are increaſing, the Sun is ſhort of the Meridian; but if they are decreaſing it is paſt the Meridian. Correct either of theſe Altitudes for Dip, Refraction, Semidiameter and (if any) Parallax in Altitude.

2. If the Latitude and Declination be both North or both South, then ſubtract the Sun's Declination from Ninety Degrees the Remainder is the Polar-diſtance. If the Latitude and Declination be one North and the other South, add the Declination to Ninety Degrees, the Sum is the Polar-diſtance.

3. Add together the Co-latitude, the Polar-diſtance and Co-altitude, and from Half their Sum ſubtract the Co-altitude to get a Remainder; then add together, the Co-ar. of the Co-latitude, the Co-ar. of the Polar-diſtance, the Sine of the Half Sum, and the Sine of the Remainder; Half the Sum of theſe, is the Coſine of a number of Degrees and Minutes, which being doubled, is the number of Degrees and Minutes of the Equator, which the Sun is then either ſhort of or paſt the Meridian for the time of obſervation.

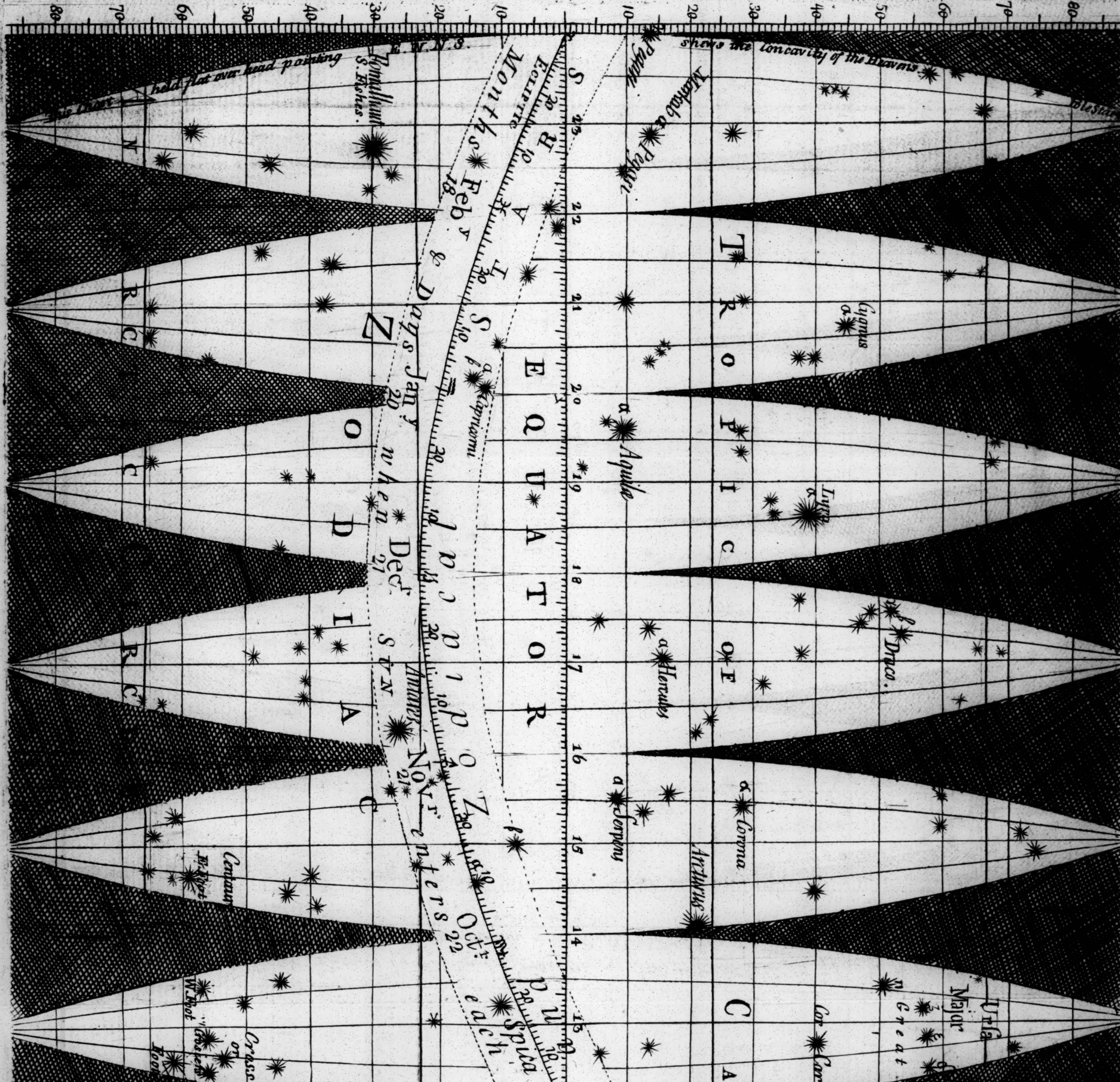
3. In taking out the Sun's Declination, it muſt be for ſomewhat near the Hour of Day or Night which it is at that time at the Place for which the Table of Declination is made. This may be known ſeveral ways. *1ſt.* By the Hour at the Place of obſervation and its imperfect Longitude. *2d.* By an obſerved Diſtance of Sun and Moon compared with one predicted for another Meridian. *3d.* By taking the Magnetic Variation, this ſhews the Longitude imperfectly by Inſpection.

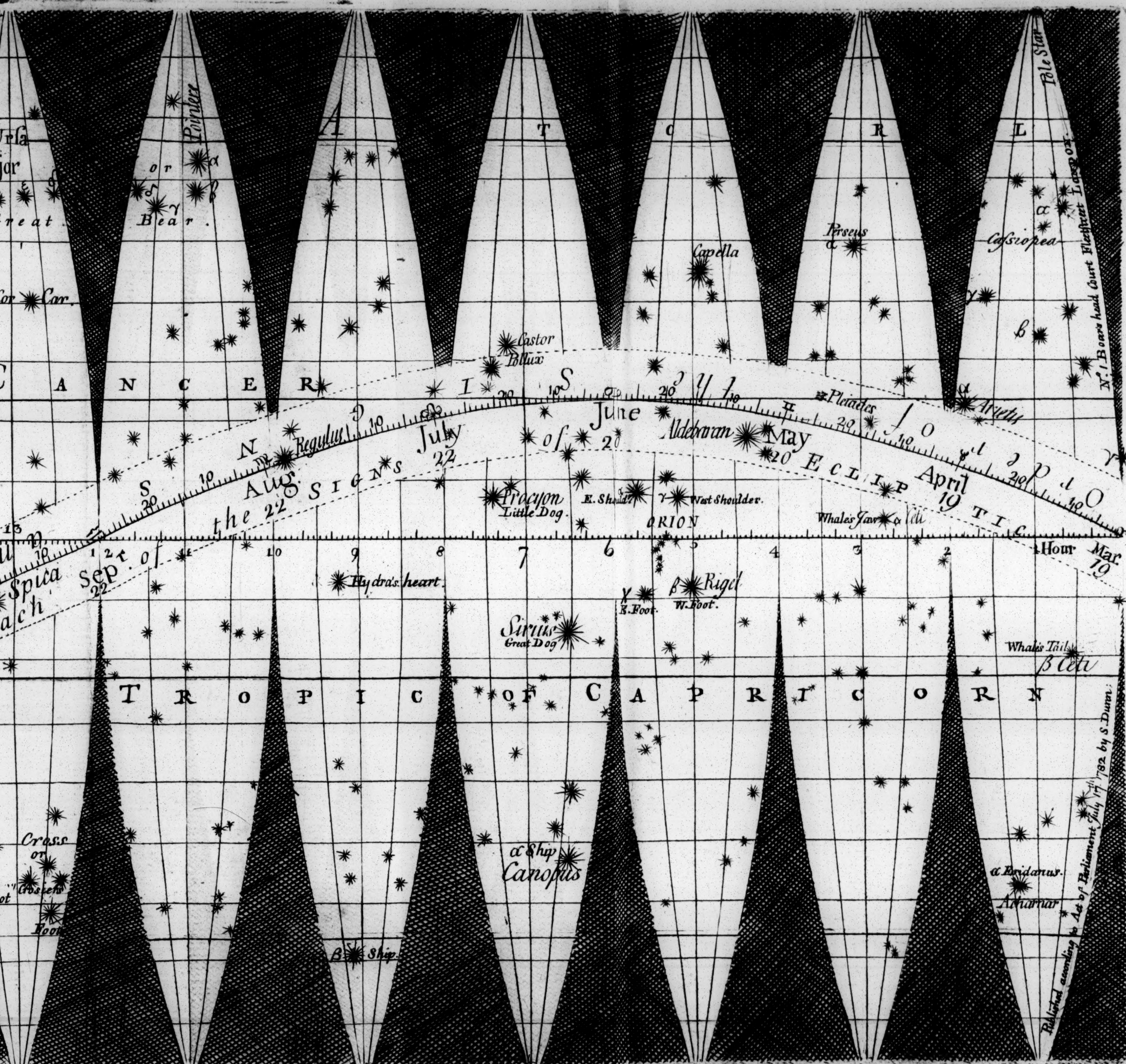
LV. *To*

A CHART of the Zodiacal Stars Used in Finding the LONGITUDE at SEA the 1st Form; By S. DUNN, Teacher of the Mathematical Sciences.

Find the Sun, Moon & Planets Places in the Zodiac, by an Ephemeris, & their Positions to the Zodiacal Stars will appear by Inspection. For Hours of a Day past Noon, count to the Left; for before Noon to the Right of the Sun, gives the Stars on the Meridian: For farther see Treatise Longitude at Sea.

S. Dunn inv. & delin.



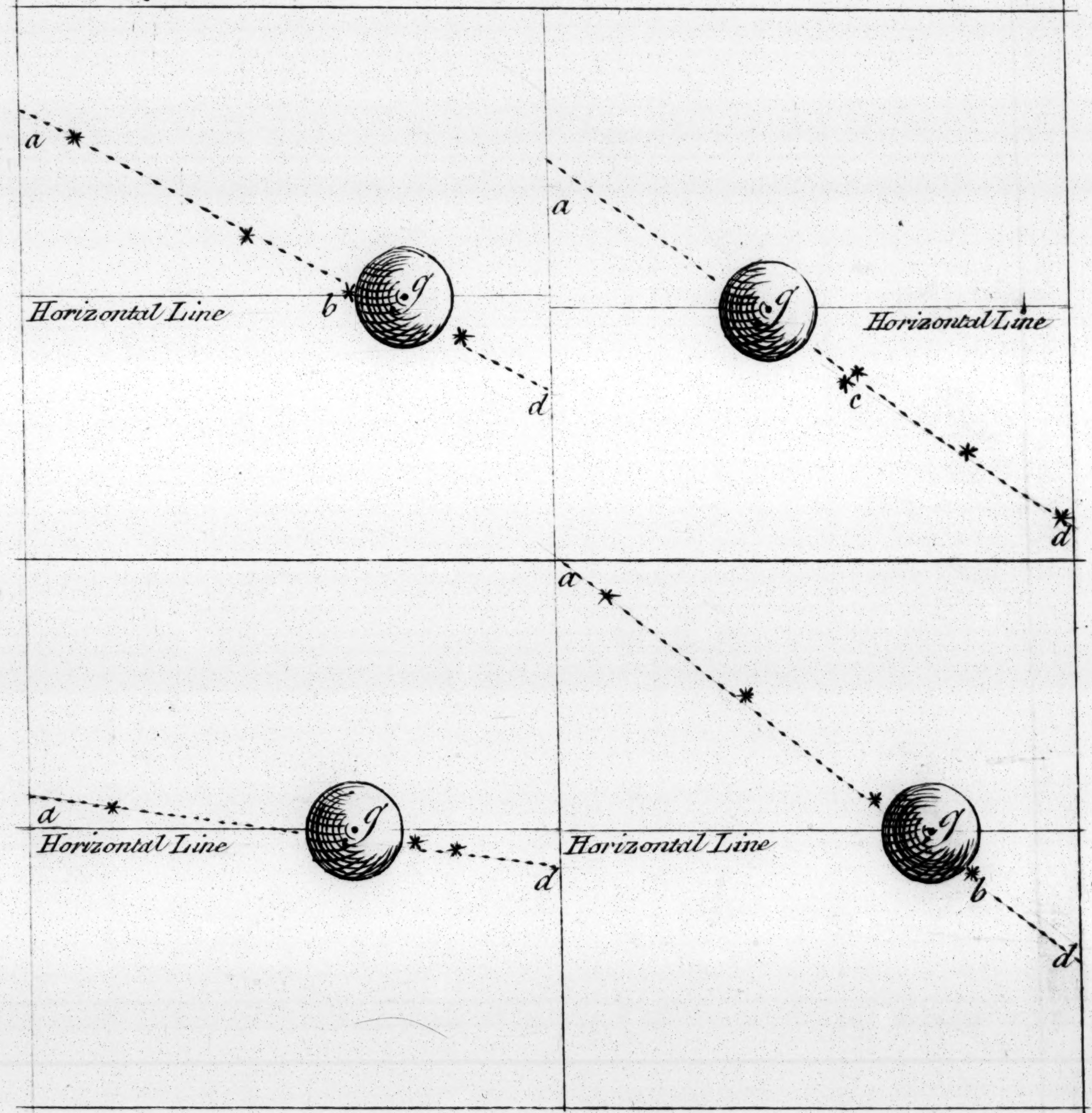


Ursa Major
Pointers
Great Bear
Cor Car.
Pole Star
Cassiopea
Perseus
Capella
Castor
Pollux
CANCER
Pleiades
Aldebaran
Arietis
Regulus
June
May
April 19
July 22
Aug
Sep. 22
SIGNS
ECLIPTIC
Spica
Procyon
Little Dog
E. Shoulder
West Shoulder
ORION
Whale's Jaw
Ceti
Hour
Mar. 19
Hydra's heart
Rigel
E. Foot
W. Foot
Sirius
Great Dog
Whale's Tail
β Ceti
TROPIC OF CAPRICORN
Cross or Crosiers
Foot
α Ship
Canopus
β Ship
α Eridanus
Achernar
Published according to Act of Parliament July 17th 1782 by S. Dunn:
No. 1 Boars head Court Fleetstreet London.

Observ'd Appearances of Jupiter & the Satellites.

♃. The Planet Jupiter as seen thro' a Telescope of from 30 to 100 power.

a. d. The Satellites Orbits
b. Satellites near Jupiter.
c. apparent Conjunctions.



From Observations the most instantaneous Conjunctions, are determinable to less than 20″ of Time, or 5 Miles Long.^d at the Equator.

Plate 6:
Page 28.

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LV.

To take Solar Time at a Place by an Altitude of a Primary Planet, or of a Fixed Star.

1. The Primary Planets, Venus, Mars, Jupiter and Saturn, are each of them visible during a part of the Twilight, when above the Horizon, and no Clouds, and so are the largest Fixed Stars; at such times and also when the Moon illuminates the Horizon, their Altitudes can frequently be taken.

2. The Declinations of these Planets, are near Truth in the Predictions; those of the Fixed Stars are correct, except a Change of near Twenty Seconds which may in some Cases arise from the Aberration of Light and Nutation. Therefore,

3. When the Altitude of a Planet or a Star has been taken, the Calculation is to be made as before for the Sun, and the Result is the Planet or Star's Equatorial Distance from the Meridian, either past or short of it.

4. From the Beginning of the Equator, compare together, the Right Ascension of the Sun, the Right Ascension of the Planet or Star, and the Equatorial Distance of the Planet or Star from the Meridian, accordingly as they are either additive or subtractive, and they give the Solar Time either short of or past Noon at the Place of Observation.

5. When Altitudes of two or more Planets or Fixed Stars can be taken, the Time may be found from each, and the Medium will be more exact.

LVI.

To find the Amplitude of the Sun or of a Star or Planet; having the Latitude and Declination.

1. Amplitude is the number of Degrees and Minutes on the Horizon, which the Sun or a Star rises or sets to the northward or southward from the true East or true West; and it is of the same Name as the Declination, whether north or south.

2. As the Cosine of the Latitude to Radius, so is Sine of the Declination, to Sine of the true Amplitude. This compared with the observed Amplitude gives the Variation of the Compass or Magnetic Needle.

LVII.

To find the Azimuth of the Sun or of a Star or Planet; having the Latitude and Declination.

1. Azimuth is the number of Degrees and Minutes on the Horizon under the Sun or a Star, which it is from the true North Point of the Horizon in north Latitude, and from the true south Point in south Latitude.

2. Get the Polar-distance as before; then, Add together the Co-latitude, the Co-altitude and Polar-distance, and from their Half Sum subtract the Polar-distance to get a Remainder. Then, add together, the Co-ar. of the Co-latitude, the Co-ar. of the Co-altitude, the Sine of the Half Sum and the Sine of the Remainder; Half the Sum of these, is the Cosine of an Arch, which being doubled, is the true Azimuth from the true North in north Latitude, and from the true South in south Latitude.

3. The true Azimuth compared with the observed Azimuth, gives the Variation of the compass, or Magnetic Needle.

LVIII.

Of the various Methods which have been proposed and applied; in order to a Discovery of the Longitude at Sea.

1. There have been no less than Twenty Methods proposed, for finding the Longitudes of Places on Land and at Sea; namely, *1st.* By placing Hulks of Ships at equal Distances across the Ocean, from which Signals were to be given by throwing up Fireworks, and making a great Report or Sound. *2d* By an Air Glass for keeping Time. *3d.* By a Quicksilver Glass for the like purpose. *4th.* By the Sun's Separation from the Fixed Stars. *5th.* By the Planet Mercury's separating from the Sun. *6th.* By the Separation of Saturn, Jupiter, Mars and Venus, from the Sun. *7th.* By the Separation of Saturn, Jupiter, Mars and Venus, from the Fixed Stars. *8th.* By Saturn's Satellites. *9th.* By the Dipping Needle. *10th.* By Jupiter's Satellites. *11th.* By Solar Eclipses. *12th* Lunar Eclipses. *13th.* By the Moon's Change in Declination. *14th.* By the Magnetic Variation. *15th.* By Timekeepers. *16th.* By a Ship's Reckoning at Sea; and by the Positions and Distances of Places on Land. *17th.* By the Moon's daily Increase in Right Ascension from the Sun. *18th.* By the like from the Fixed Stars. *19th.* By the Moon's angular Distance from the Sun. *20th.* By the Moon's angular Distances from the Fixed Stars. Some have pretended to the Discovery of a Perpetual Motion, and that such would be assisting for the Longitude, without considering that a constant Weight applied to a Machine, with either a small Addition or Subduction, as an Allowance for the Change of Gravity from one place to another, would answer the same use.

2. Of these Twenty Methods, the three first have been exploded. It is but a few Years since five large Meteors were seen to pass through the Air in the interval of about ten Weeks, one of these was observed by me and others, at the Dis-

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2. Get the Polar-distance as before; then, Add together the Co-latitude, the Co-altitude and Polar-distance, and from their Half Sum subtract the Polar-distance to get a Remainder. Then, add together, the Co-ar. of the Co-latitude, the Co-ar. of the Co-altitude, the Sine of the Half Sum and the Sine of the Remainder; Half the Sum of these, is the Cosine of an Arch, which being doubled, is the true Azimuth from the true North in north Latitude, and from the true South in south Latitude.

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2. Of these Twenty Methods, the three first have been exploded. It is but a few Years since five large Meteors were seen to pass through the Air in the interval of about ten Weeks, one of these was observed by me and others, at the Dis-

tance of some Miles from each other; by these Observations, the Meteor was many Miles high in the Atmosphere. At that time Balloons had not been heard of in London; since, many surprizing ærial Voyages have been made by them, but they have not been yet applied to either geographical or nautical Subjects.

3. The Discovery of the Longitude on Land by the Sun's Separation from the Fixed Stars, is one of the most antient Methods that has been known, but it is embarrassed with such Difficulties that it can never be practised with any Accuracy that will make it either of general or particular Use. It allows at a Medium, about four Minutes of Time to three hundred and sixty Degrees of Longitude; or a Second of Time to a Degree and Half of Longitude; whilst, Experience proves that neither correspondent Transits nor single ones can be easily taken to that Accuracy, whereas the Longitude of a Place is required ten times as exact on Land, and at Sea this Method is wholly impracticable.

LIX.

Of Longitude by the Primary Planets separating from the Sun.

1. The Planet Mercury is so near the Sun, and his periodic Time is so short, that at his inferior and superior Conjunctions with the Sun, his apparent Separation has been by some, thought sufficient for the Longitude. But even then it is much too little; besides, it is a Question, how correctly his Place can be predicted? His apparent change of Place is very irregular. It is but seldom he can be observed on account of his Proximity to the Sun; and the times of his appearing are such, that no Instrument now in Use, is adapted for taking his Altitude or angular Distance from the Fixed Stars, except in very favourable Weather, and such as but seldom happens.

2. The Planets, Venus, Mars, Jupiter and Saturn, may often be observed, but there is the same objection concerning the Accuracy of the Tables which give their Predictions, and a much greater objection to their small daily Separation from the Sun and Fixed Stars. Therefore,

3. The best Uses of these Planets, will arise from their Declinations and Right Ascensions, when these can be predicted somewhat near the truth; for then, they may be applied in taking the Latitudes of places, like the Sun and Fixed Stars.

LX.

Of Longitude by Saturn's Satellites.

1. These cannot be observed but by the most able Observers and the use of the best Telescopes, under favourable circumstances of Weather and the Planet's Position in the Heavens.

LXI.

Of Longitude by the Dipping Needle.

1. In this Method, the Instrument for taking the Dip must be made with all possible Skill and Care, otherwise the Observations cannot be depended on; and then, altho' in some places the Dip will differ considerably, within the Limits of a small Difference of Longitude, in many other places it will be nearly the same for a considerable Distance near the same Parallel of Latitude.

2. The Writers on this Method have supposed certain Magnetic Poles to which the Needle points under due Regulations; but this agrees with no Observations of the Dip on the Earth or Seas.

LXII.

Of Longitude by Jupiter's Satellites.

1. Jupiter's Satellites, are four Moons moving round him in their respective periodic Times. By reason of the Length of Jupiter's Shadow and the Planes of their Orbits being near to that of Jupiter, they frequently pass through his Shadow, are thereby eclipsed, and their Eclipses may be observed at different Places on the Earth.

2. The Immersions and Emersions of these Satellites, furnish Observers with frequent Opportunities for determining the Longitudes of places on Land, by correspondent Observations; especially those of the first Satellite on account of its Splendor and swift Motion into and out of Jupiter's Shadow. See Figure, Jupiter's Satellites.

3. In making these Observations, a person should use a Telescope magnifying not less than Thirty times, and a Pendulum Clock beating either Seconds or Half Seconds of Solar Time.

4. When two Persons at different Places on the Earth, have observed either the Immersion or Emersion of the same Satellite and compared their Times; the Difference, is the Difference of Longitude of the Places where they observed.

5. The Tables for predicting the Times of the first Satellite, are not much different from the Truth; therefore, Predictions made by them, may be used as one Observation; and a single Observation beside, gives the Longitude.

6. This Method is excellent for determining the Longitude on Land; but cannot be applied often at Sea, by reason of the Motion of a Ship.

LXVIII.

Of Longitude by Solar Eclipses.

1. Because a Solar Eclipse is occasioned by the Moon's

METHODS FOR LONGITUDE. 31

Moon's Interpoſition between the Sun and Earth, and the Moon's daily Receſſion is from Weſt to Eaſt; therefore, a Solar Eclipſe begins on the Sun's Weſtern Limb, and ends on the Eaſtern Limb, and nearly in the direction or poſition of the Ecliptic at that time, in the Heavens.

2. If the Moon's Diſtance from the Earth was great enough to annihilate the Moon's Horizontal Parallax, this Method would be preferable to all others for ſettling the Longitude of Places on Land, and at Sea, whenever thoſe Eclipſes could be obſerved; for, their Beginnings and Endings may be taken to great Exactneſs, and the Difference of Times would then be the Difference of Longitude. But,

3. By reaſon of the Moon's Horizontal Parallax, a Solar Eclipſe that is total at one place may be partial at another, and at other places may be none. This allowance for the time whilſt the Moon is paſſing to one or another part of the Sun's Limb, or removing from it, is not to be found without a difficult proceſs, and ſuch as at leaſt is liable to Exception.

4. On theſe accounts, Solar Eclipſes cannot be obſerved and applied with that eaſe and certainty that is deſirable. But, the circumſtances attending a Solar Eclipſe, may ſometimes admit of removing theſe Difficulties (theſe depend on the Poſitions of the Circles of the Sphere and the quantity of the Moon's Horizontal Parallax) and then it may be applied for the Longitude.

LXIV.

Of Longitude by Lunar Eclipſes. See Map of the Moon.

1. Of all the Methods that have been propoſed for finding the Longitudes of places either on Land or at Sea, that are not general, this is the compleateſt; for, it is encumbered with no Parallaxes, it is moſt eaſily to be obſerved, and admits of more Obſervations than any other Phænomenon whatever.

2. In a total Lunar Eclipſe, if the Ecliptic happens to be nearly perpendicular to the Horizon weſtward, the Eclipſe begins at the upper Limb of the Moon, and ends near the lower Limb; but, if perpendicular eaſtward, it begins at the lower Limb, and ends near the upper Limb. In other Poſitions of the Ecliptic, it begins on the Eaſt Side and ends on the Weſt Side of the Moon.

3. Before the Eclipſe begins, for a conſiderable time, the parts from Ariſtarcus to Kepler and Copernicus, are affected with a faint Shade, which goes on toward Langrenus, the oppoſite part where the Eclipſe is to end; this Shade is the Penumbra.

4. In ſuch an Eclipſe, the real Shadow firſt touches the Moon near Ariſtarcus, and comes on with a Velocity of near two Minutes of a Degree in four Minutes of time; and when the Limb firſt loſes its perfect Roundneſs, is the Beginning of the Eclipſe. If this be obſerved by the naked Eye and alſo by a Teleſcope magnifying Sixty times, the former will perceive it a Minute of Time ſooner than by the latter; during this Interval the Shade has moved but Half a Minute of a Degree.

5. As the Diameter of the Shade is much larger than that of the Moon, the Arch of the Shade will be flatter than the Moon's Circumference and touch the Spots on the Moon's Face accordingly; theſe are principal Sights to be taken and wrote down as they happen.

6. On Land, theſe Obſervations will be greatly aſſiſted by the uſe of either a Pendulum Clock beating Seconds or Half Seconds of Solar Time, and previouſly ſet to it with the greateſt care. At Sea a good Pocket Watch that may be depended on for keeping ſuch Time, will anſwer the like purpoſe; the Times with the names of the Spots when touched, biſected, and uncovered, to be noted and remain unaltered.

7. If no Time-keeper be at hand, and the Latitude of the place is known, in favourable Weather the Altitude of a known Fixed Star, or a Primary Planet, may be taken, and from theſe the Time may be calculated; but, when a number of Obſervations are to be made, it will be beſt to have a good Time-keeper.

8. At the End of the Eclipſe, the oppoſite Limb of the Moon near Langrenus is affected as at the Beginning, until the Moon's Face receives her former Luſtre. In ſuch an Eclipſe, the Beginning and End of Total Darkneſs may be eaſily and correctly taken, theſe will be great additions; and the Mediums of the Touches, Biſections and Uncoverings of the Spots will be their central Touches by the Shade.

9. In Lunar Eclipſes that are Partial, the Shadow ſometimes paſſeth over the northern and ſometimes over the ſouthern part of the Moon. In ſuch Eclipſes, the ſame method of Obſervation is to be obſerved, as in thoſe that are total.

10. When ſuch Obſervations have been accurately made, by two or any other number of Perſons, however remote from each other, the difference between either the Beginning, End, or Touch of any Spot, in Solar Time at one Place, compared with that af another Place, is their Difference of Longitude. But, if the predicted Beginnings or Ends be uſed inſtead of Obſervations, one Obſerver may anſwer the like purpoſe, though not ſo exact.

LXV.

Of Longitude by the Moon's Change in Declination.

1. Seeing that the Moon's diurnal Receſſion from Weſt to Eaſt, or rather her daily Preceſſion in the Zodiac is ſo great, and the plane of her Orbit is inclined to that of the Earth; when ſhe happens to be near either of the Equinoctial Points, her daily Change in Declination amounts to at leaſt Half her daily Receſſion. Theſe Circumſtances have been thought by ſome Perſons, the more favourable, as her Declination can be frequently taken without conſiderable Error, both by Altitudes from the Horizon and by her Poſitions to the Fixed Stars.

2. In this Method, the original Tables muſt give the Moon's Latitude to the greateſt Exactneſs; the Declination out of the Meridian is encumbered with Parallaxes and Refractions; the apparent Horizon is both variable and affected with the Earth's Spheroidical Figure; the Declination itſelf, when the Moon is near the Tropics, changes very ſlowly; and ſome of theſe Difficulties can never be ſufficiently overcome, for general Uſe and eaſy Practice to the Accuracy that is wanted. That the Longitude at Sea may be taken by it in an imperfect manner is certain, and that the Latitude may be taken by it to any Accuracy that is uſeful in Navigation.

LXVI.

Of Longitude by the Magnetic Variation.

1. In former Ages, after it became known that the Variation of the Magnetic Needle was a real Property, not depending on any particular Conſtruction of the Compaſs, Navigators and Mariners were not long at a loſs how to apply it in determining (at leaſt to ſome degree of accuracy) the Place of a Ship at Sea, in order thereby to avoid Dangers and ſhorten their Voyages. At thoſe times, by repeated Obſervations, they found that the Variation was different at different Places, and that when they came to ſuch a Latitude and had ſuch a Variation of the Compaſs, their Courſe and Diſtance to ſome near Land was thereby known, and would on different Voyages anſwer accordingly.

2. Afterward it was diſcovered that the Variation altered at the ſame Place, and that the Compaſſes would at ſome particular Places, be ſuddenly affected with a Change unexpected; but, notwithſtanding this, the Subject was deemed of that Importance in Navigation, about the beginning of the Seventeenth Century, that at great Expences Men were employed and proper Ap-

plications made for diſcovering the Laws of this ſurprizing Syſtem. But, in bringing the beſt Materials together that could be collected, ſome were new others obſolete, and ſuch, that the then State of this Syſtem, was but imperfectly ſhewn.

3. Near Half a Century after, another elaborate Undertaking of the ſame kind was begun and carefully executed; but, in ſuch a manner as to give no Inſight concerning the Laws of this Syſtem. About Twelve Years after, the ſame work was republiſhed with ſuppoſed authentic Improvements; nevertheleſs, it is now certain that, both its Stages had Errors to the amount of many Degrees of Variation, and over Seas the moſt frequented for many Ages, by Navigators of all Nations.

4. The Variation Charts of the Magnetic Needle, which have been publiſhed by me, have been chiefly from my own Improvements in this Science, and without theſe, they never could have been ſo correctly drawn as they are. As able Nautical Aſtronomers as are in this or any other Kingdom have experienced their great Utility, and found the Longitude by them, at Sea; and as accurately as by any other Method whatever, at Places where the Variation Lines have not been ill adapted for that purpoſe.

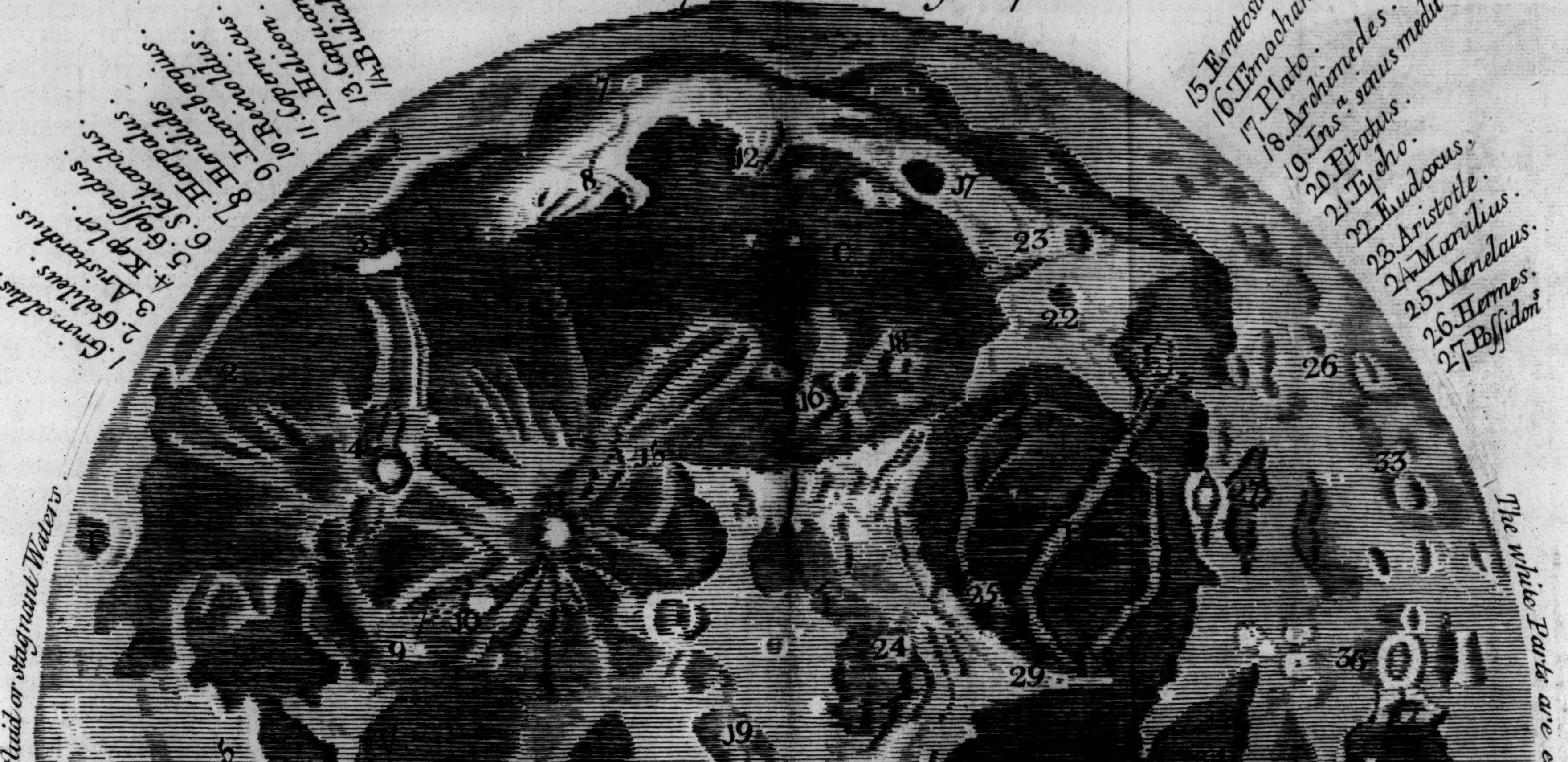
5. In uſing them, the Bearing of either the Sun, a Planet or Star is taken by the Compaſs Card, and at the ſame time an Altitude. Here, having got the Polar-diſtance of the Sun, Planet or Star, add together the Co-latitude, the Co-altitude and Polar-diſtance, and from their Half Sum ſubtract the Polar-diſtance to get a Remainder. Then, add together, the Co-ar. of the Co-latitude, the Co-ar. of the Co-altitude; the Sine of the Half Sum, and the Sine of the Remainder. Half the Sum of theſe four is the Coſine of an Arch, which being doubled, is the true Azimuth from the North in North Latitude, and from the South in South Latitude. This compared with the Magnetic Bearing or Azimuth gives the Variation.

6. With the Latitude and Variation, enter the Variation Chart; and in their Angle of Meeting is the Longitude, when the Variation Lines on the Chart run near enough to the North and South, and to each other, for that Purpoſe; but, in other Caſes or in thoſe Seas where they run nearly Eaſt and Weſt and at wide Diſtances from each other, the Longitude cannot be determined by them; nevertheleſs, at ſuch Places they may ſometimes be applied for determining the Latitude having the Longitude and Variation; and in all Caſes whatever, they may be well applied for knowing the magnetic Courſe, when the Latitude and Longitude are known by the Ship's Reckoning.

LVIII. Of

A MAP OF THE MOON,

with the Names of the dark and light Spots



In a Lunar Eclipse, the Solar Time when the Shadow touches any of the Spots, is to be taken as correctly as possible at two distant Places, and the Difference of those Times is the Difference of Longitude of the two Places.

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LXVII.

Of Longitude by the Ship's Reckoning.

First general Method of finding Longitude at Sea.

1. In this Method of finding the Longitude at Sea, it is neceſſary to underſtand all the parts of Practical Navigation, the Methods of inferring the Longitude from the Courſes and Diſtances ſailed; the taking of the Latitude by Meridian Altitudes, and the keeping an exact Account of theſe and other Particulars relative to them, at Sea, from the time a Ship ſails to the time of her Arrival at a deſigned Port.

2. In this Method, nothing is to be done but what agrees with the Theory of Navigation and the nature of Loxodromic Lines, the Approaches of the Meridians and the Diminution of the Degrees of Longitude, from the Equinoctial toward the Poles. Mercator's Sailing amply provides for all theſe things (except for the irregularity of the Earth's Figure) therefore, it is much eſteemed. Middle Latitude Sailing, when the Diſtances are ſhort comes near the Truth, but in long Diſtances it is erroneous.

3. In order to correct the many Errors that muſt ariſe in keeping an Acconnt of the Difference of Latitude and Longitude, it is proper to apply ſuch a Method as will effectually prevent thoſe Errors from ariſing and entering into the Calculation; this may be done as follows. *1ſt.* Let all the Courſes ſailed by Compaſs be carefully taken, and the Diſtances ſailed, carefully meaſured from the Beginning to the End of every Day. *2d.* From theſe, find the whole Magnetic Difference of Latitude and Magnetic Departure, and thereby, the Diſtance ſailed made good, and the Magnetic Courſe made good. *3d.* Correct the Magnetic Courſe made good, by the true Mean Variation, to get the true Courſe made good, with which and the Diſtance made good, get the true Difference of Latitude and true Departure. *4th.* With theſe, proceed to get the true Latitude and Longitude, as in the uſual Practice of Navigation. Or otherwiſe, more expeditiouſly and correctly by the Formula, Inſtructions and Journal publiſhed lately by me for thoſe purpoſes.

4. If theſe Methods are uſed, there can be but little (if any) Error whatever in the Concluſion, from a want of knowing even the true Variation of the Compaſs, at any time or place; for, the nominal Variation of the Compaſs here applied, will always be that taken by the Compaſs, and whether that be true or falſe, from the Needle's Situation to the Card, there will be no Difference in the true Courſe made good.

LXVIII.

Of Longitude by Time-keepers.

Second general Method of finding Longitude at Sea.

1. In this Method, the Time-keeper to be uſed is ſuppoſed to be ſo adjuſted before it goes to Sea, that it will either keep Mean Solar Time; or otherwiſe, that the Number of Seconds it gains or loſes thereon per day, are known. In the latter Caſe, the Number of Days multiplied by thoſe Seconds, gives the whole gain or loſs, from which the Solar Time at the place ſailed from, is ſuppoſedly known. Then, having an Altitude of the Sun, or of a Star, or a Planet, with the Declination, and the Latitude of the Place; the Solar Time at the Ship may be computed; and the difference of theſe two Times is the Longitude.

2. In this Method, it is neceſſary to know whether the Time-keeper keeps Equal Time or gains or loſſes thereon. After it has received Cleaning and freſh Oil; ſome Artiſts will have the Balance make larger Vibrations, and therefore, as they think, it muſt take a longer Time in vibrating; Others, of at leaſt as good Judgment, find the quite contrary. It is farther obſerved, that, the Alteration when it begins, is not regular nor proportionate, and the longer it goes the greater is the Uncertainty.

3. Under ſuch Circumſtances, when the Machine has gone long and is removed to a diſtant Meridian, no Obſervations depending on the apparent Diurnal Motion, can ſhew to what the whole accumulated Error amounts; nor any thing leſs than the true Longitude itſelf that has been made (which is the Thing ſought) and therewith proper Celeſtial Obſervations. But,

4. Where a Time-keeper is perfect, the Method of finding Longitude by it is thus. Take an Altitude of the Sun, a Planet or a Fixed Star, and at the ſame time note the Hour, Minute and Second, as ſhewn by the Time-keeper. Having the Declination, get the Polar-diſtance; then, add together the Co-latitude, the Polar-diſtance and Co-altitude, and from their Half Sum ſubtract the Co-altitude to get a Remainder. Then, add together, the Co-ar. of the Co-latitude, the Co-ar. of the Polar-diſtance, Sine of the Half Sum, and Sine of the Remainder; Half the Sum of theſe four is the Coſine of an Arch, which being doubled is the Time paſt Noon in Degrees and Minutes for an Afternoon Obſervation; but, ſhort of Noon for a Forenoon Obſervation, at the Ship. Turn the Time Solar per Time-keeper into Degrees and Minutes; then, in an Afternoon Obſervation, ſubtract one Time from the other, the Remainder is the Longitude Weſt when the Time-keeper's Time

is greateſt, but otherwiſe Eaſt Longitude. In a Forenoon Obſervation, add together the two Times and their Sum is Weſt Longitude.

LXIX.

Of Predictions for the Longitude at Sea.

1. Predictions of this kind, are the Reſults of Calculations made from Tables conſtructed according to the beſt Theories of the Celeſtial Motions and according to Obſervations made with the moſt perfect Inſtruments that can be uſed. Such Inſtruments and the continual Application of them in obſerving the Motions of the Sun, Moon, Primary Planets and Fixed Stars, requiring a great Expence, they are therefore at but few places except where the Munificence of Princes, or Perſons of great Fortune have placed them for making Diſcoveries and Advancement in the Aſtronomical Sciences. Therefore,

2. Such Inſtruments are the principal Furniture of Aſtronomical Obſervatories in different Countries. The Angular Inſtruments in thoſe places, are large, fixed to firm walls, and, if moveable in the Plane of the Meridian, are conſtructed accordingly, ſo that they have great Advantages over ſmaller Inſtruments that are portable. The Clocks have all Advantages that can be given them. The chief end and deſign of theſe Inſtitutions is to obtain an accurate Knowledge of the Celeſtial Motions and make them ſubſervient to the Improvement and Perfection of Geography and Navigation.

3. In ſuch Obſervatories, the Times when the Celeſtial Bodies tranſit the Meridian, are accurately obſerved; thoſe Obſervations and the Laws of the Solar Syſtem are applied together in a mathematical way, and hence Tables are made for predicting the apparent Returns of the Sun, Moon, Primary Planets and Fixed Stars, for times to come.

4. From thoſe Tables, annual Tables or Ephemerides are formed, in which the Right Aſcenſions, Declinations, and other Poſitions of the variable Phænomena, are inſerted for every Day in the Year. In ſuch are ſhewn, *1ſt.* The Right Aſcenſion Declination, Longitude and Semidiameter of the Sun. *2d.* The Right Aſcenſion, Declination, Longitude and Latitude, Horizontal Parallax and Semidiameter of the Moon. *3d.* The Right Aſcenſions, Declinations, Longitudes and Latitudes of the Primary Planets. *4th.* The Angular Diſtances (at certain ſhort Intervals of Time for ſome known Meridian) of the Sun from the Moon, and of the Moon from the Zodiacal Stars. Theſe particulars are in a continual State of Mutation, and although they may ſome of them return nearly to the like Poſitions after a long Interval of Time; from the Laws of the Syſtem, the Time of their Return to the ſame preciſe Situation is indefinite and not to be known. Theſe are the principal Predictions wanted, in all the aſtronomical Methods of finding the Longitude at Sea.

LXX.

Of Longitude by the Moon's Change in Right Aſcenſion from the Sun.

1. In this Method, the Lunar Tables uſed in predicting the Moon's Longitude, are required to be as correct as for any Method whatever, and from theſe predicted Longitudes, the Moon's correct Right Aſcenſions from Day to Day, or for parts of a Day, are to be inſerted in Tables for ſome Firſt Meridian, for uſe at Sea.

2. At Sea, having the Sun's Altitude, the Moon's Altitude, and the Angular Diſtance of Sun and Moon, together with the other Requiſites, the Right Aſcenſion of the Moon may be found for the time of Obſervation; and conſequently, the Hour, Minute and Second of that inſtant for the Firſt Meridian.

3. Having the Latitude of the Place, the Sun's Declination and his Altitude, the Time at the Ship may be found; and this compared with the Time at the Firſt Meridian, gives the Longitude of the Ship. This Method is ſupplied, and in a more eaſy manner, by the three hourly Diſtances of Sun and Moon.

LXXI.

Of Longitude by the Moon's Change in Right Aſcenſion from the Fixed Stars.

1. In this Method, the ſame correctneſs of the Tables is required for the Stars and Moon, and nearly the like Obſervations to be made as for Sun and Moon, until the Star's Diſtance from the Meridian of the place of Obſervation is found, whether it be paſt or ſhort of it. Then,

2. Having the Sun's Right Aſcenſion, the Star's Right Aſcenſion, and the Star's Diſtance from the Meridian, the Time at the Ship may be found; and this compared with the Time at the firſt Meridian, gives the Longitude of the Ship. This Method is ſupplied, and in a more eaſy manner, by the three hourly Diſtances of the Moon and Zodiacal Stars.

LXXII.

Of taking and preparing the Moon's Angular Diſtance from the Sun.

1. This Method and the following (which is on the ſame Principle), are the two Methods which

which are now practised with Success by Nautical Astronomers.

2. When Observations are to be made for the Longitude by Sun and Moon, it is convenient to have three Persons, one for the Distances, another for Sun's Altitude and another for Moon's Altitude. The first should have a Sextant adjusted to the greatest Exactness, the others may have Octants.

3. All three should know, that they are to make their respective Observations as nearly at equal Intervals of Time as possible, and each as near the same Instant as they can. The person who takes the Distances is the principal and leading Person, and should speak when he has joined the Limbs; at the same Instant, the others should be ready to take off and write down their Altitude of Sun's lower and Moon's lower or upper Limb.

4. When several such Distances and Altitudes have been taken and written down, each Observer is to add up his own, and divide their Sum by the Number that have been taken, and the three respective Mediums will be the co-temporary Observations. When the greatest Accuracy is required, the Distances should be farther corrected by the foregoing Method of Substitution.

5. The Distance is now prepared, but for each Altitude, it should be noted what Tables are to be next used; if the Linear Tables, subtract the Dip and add the Semidiameter of Sun or Moon's lower Limb, but subtract the Dip and Semidiameter for Moon's upper Limb, this clears the Altitudes, and prepares for Calculation. If the Method by Sun and Moon's or by Star and Moon's Angles is to be used, the Refraction also is to be subtracted.

LXXIII.

Of taking and preparing the Moon's Angular Distance from the Fixed Stars.

1. Here is the same Method of Preparation and Observation, the same number of Observers and the same Instruments as before. The Moon's enlightened Limb has its Image brought to the Star. The most favourable Opportunities (all other things suitable) are during the Twilights and when the Moon's Light renders the Horizon visible.

2. The Star's Semidiameter is nothing. If the Horizon be imperfectly defined, it is best to observe when the Star is as nearly East or West as possible (because an Error in the Star's Altitude chiefly affects the Time at the Ship, and that will thereby be greatly removed) in other Cases, there are various ways of supplying the want of a Horizon sufficiently correct for this purpose. This prepares the Star and Moon for Calculation.

LXXIV.

Principles, geographical and hydrographical, concerning the prime or first Meridian, from which Longitude begins to be reckoned, and how it is formed.

1. Geographers and Hydrographers, frequently name the whole Great Circle passing round the Earth and Seas from north to south, the Meridian of a Place through which it also passeth; but, it seems more accurate to name it that Great Semicircle which passeth through the Place and terminates at the Earth's Poles.

2. According to this Definition, the Meridian of London passeth through St. Paul's Cathedral London, and being continued northward, leaves England near Flamborough-head in Yorkshire; it then crosseth the Northern Seas, and being continued Thirty-eight Degrees and twenty-nine Minutes from London, comes to the Earth's north Pole.

3. This meridian being continued southward, leaves England near Brighthelmstone in Sussex, It then crosseth the British Channel, and enters France near the Town of Fecamp and the little Village of Auberville not far from the Entrance of the River Seine. By crossing the Pyrenean Mountains it enters Spain, and leaves it a little westward of the Entrance of Ebro River. It then passeth over the Mediteranean Sea, enters Africa near Oran, and leaves it about Sixty Miles eastward from Cape Three Points on that Coast. It then enters the Ethiopian Sea, and at the distance of Fifty-one degrees, and thirty-one Minutes from London, comes to the Equinoctial Line. Here the Longitudes of Places from the Meridian of London begin to be reckoned in the Arch of the Equinoctial, eastward and westward One hundred and eighty Degrees, for East and West Longitude from London.

4. Continue this Meridian Ninety Degrees southward and it comes to the South Pole of the Earth. This Distance from the North to the South Pole, completes the first Semicircle of the Meridian of London, at all places of which, it is Noonday at the same Instant of Time.

5. Continue the first Semicircle through the Earth's Poles to compleat the opposite or second Semicircle, and it will pass through the Great South Sea, and it will become One hundred and Eighty Degrees at the Equinoctial, distant from the first Semicircle. At all Places in this latter, it will be Midnight when it is Noon-day at the former. This compleats the Great Circle commonly called the Meridian of London, dividing the East Longitude from the West.

6. The Meridian of Greenwich passeth through the Royal Observatory at Greenwich. The near-

est

eſt diſtance from St. Paul's London, to the Meridian of Greenwich, is four Miles wanting a ſixteenth of a Mile. The neareſt diſtance of thoſe Meridians at the Equinoctial, is ſix nautical Miles and a third; this ariſes from an enlargement of the Longitude Degrees toward the Equinoctial, and their being ſhorter toward the Poles. Hence, at the Equinoctial a Degree of Longitude is a Degree of Diſtance, and a Minute of Longitude is a Mile of Diſtance. Conſequently,

7. Almoſt all Europe, Aſia and Africa, are in Eaſt Longitude from London and Greenwich. North and South America, are in Weſt Longitude from London and Greenwich. The Ocean's and Seas northward of Europe and Aſia, the Baltic and Caſpian Seas, almoſt all the Mediterranean, part of the Ethiopic Ocean toward the Coaſt of Africa, the Indian Ocean, the China Seas, and part of the Pacific Ocean, theſe are in Eaſt Longitude from London and Greenwich. The Atlantic or Weſtern Ocean, the Southern Ocean toward South America, and a great part of the Pacific Ocean, theſe are in Weſt Longitude from London and Greenwich.

8. The firſt Meridian hath been ſuppoſed by the Britiſh Nation to be that of London for long time paſt, probably becauſe it is the Metropolis of the Kingdom; other Nations have had their firſt Meridians at their Capital Cities; but, ſince Aſtronomical Obſervatories have been erected at different places throughout Europe for the expreſs purpoſes of improving Geography and Navigation, theſe firſt Meridians have been ſuppoſed to begin at ſeveral of theſe Obſervatories reſpectively, and the Longitudes of places have been reckoned from them accordingly. This is particularly to be noted, in reading the Predictions that are in Ephemerides and other Tables of the Places of the Celeſtial Bodies, becauſe ſuch are underſtood to be for the Times at thoſe Meridians for which the Calculations have been made, and for no others.

LXXV.

Of the Meaſurements of a Degree of Latitude, at different times and places on the Earth's Surface; and its Application to the Dip of Horizon.

1. Although the firſt Principles of the Mathematical Sciences were known to the ancient Aſtronomers, yet they made uſe of ſuch imperfect Methods for determining the true Magnitude of the Earth and Seas, that their Concluſions could hardly be otherwiſe than imperfect.

2. Hiſtorians relate that, Anaximander about 550 Years before Chriſt, meaſured, and deduced the Circumference of a Terreſtrial Meridian, but that he made it very different from what it is now found by indiſputable Obſervations. That,

3. Eratoſthenes, about 200 Years before Chriſt meaſured, and found it nearly as it is at preſent.

4. Poſſidonius, a little before Chriſt, obſerved the Meridian Altitudes of the Star Conopus (in the Conſtellation of the Ship) both at Rhodes and at Alexandria, and found the Circumference of a Meridian.

5. The Mathematicians of Maimon King of Arabia or Caliph of Babylon, meaſured two Degrees, in the Plains of Meſopotamia, and found the Circumference of a Meridian nearly as it had been found by Eratoſthenes, and ſomewhat larger than it had been found by Poſſidonius.

6. About the Year of Chriſt 1550 Fernelius meaſured from Paris to Amiens. In 1618 Snellius meaſured at Alkmaer, at Bergen-op-zoom, and at Leyden. In 1635 Richard Norwood meaſured from London to York. In 1671 Picard meaſured between Malvoiſine and Amiens. In 1718 Caſſini, between Dunkirk and Collioure. In 1736 and ſince that time, Meaſurements have been made at the north Polar Circle nearly; near the Equinoctial Line in South America; in Italy, Germany, and other Places.

7. In 1687 Sir Iſaac Newton publiſhed his Principia, wherein he determined from the Laws of Gravitation, the Centrifugal Force ariſing from the Earth's Rotation and the Meaſures of a Degree of Latitude, as they had been made before his time, that the Diameter of the Earth's Equinoctial Circle was longer than its Polar Axis by Thirty-four Miles and a fifth. Since his time others have computed after a manner but little different from his, and found that Difference ſix or ſeven Miles more. Others have concluded the Difference of Diameters to be greater, but without good Foundation; for, if the above Difference be admitted and due allowance be made for the Deviation of Gravitation from a direction toward the Earth's centre (as has been before particularized) the actual Meaſurements will not differ much therefrom.

8. Having the Meaſure of a Degree of Latitude, near the Middle of the meridional Quadrant, the Semidiameter of the Earth and Seas at that Part is known; and from the ſmall Increaſe of that Semidiameter toward the Equinoctial, and ſmall Decreaſe toward the Poles, in compariſon with the Whole and with any Heights on Land or at Sea, the one is much greater than the others.

9. In the Plane of any Meridian of the Earth, ſuppoſe a Tangent Line drawn north and ſouth at Contact with the Equinoctial Line, and through every Minute of Latitude on that Meridian, and from

from the Earth's Centre, let there be drawn ſtraight Lines to meet the Tangent Line; ſo will the former be natural Tangents, and the latter Natural Secants, to the Radius of the Equinoctial, for every Minute of Latitude. Then, ſuppoſing a Perſon elevated to any Height above the Earth's Surface at the Top of a Secant, an Arch of the Meridian would appear to him, from the Equinoctial to the point on the Earth's Surface under him; and the number of Minutes of Latitude, would be thoſe of that Secant to the Earth's Radius.

10. This ſuggeſts an eaſy Method of finding, not only the Allowance commonly made for the Dip of Horizon, when the Eye is elevated any number of Feet above the Surface of the Sea; but alſo, the Diſtance that will appear on the Curvilinear Surface of the Ocean, at a given Height, when the Difference of the Earth's Equinoctial Diameter and polar Axis is known, as the Radius of Curvature will then be alſo given.

11. The Length of a Degree of Latitude to each ten Degrees of Latitude, by Sir Iſaac Newton, was thus.

Deg.	Toiſes.	Deg.	Toiſes.
0.	56636.	50.	57074.
10.	56659.	60.	57196.
20.	56724.	70.	57295.
30.	56823.	80.	57360.
40.	56945.	90.	57382.

The Toiſe, is Seventy-ſix Engliſh Inches and Seventy-one hundredths of an Inch; and therefore (according to theſe Meaſures) the Length of a Degree of Latitude, is not Half a Mile ſhorter at the Equinoctial, nor Half a Mile longer at the Poles, than in the Middle of the Temperate Zones.

LXXVI.

Of the Tables for ſhewing the Dip, or Depreſſion of the Viſible Horizon, below the Horizon of the Sea.

1. Could the Eye of an Obſerver be coincident with the Surface of the Sea, when the Waters are undiſturbed by any Force or Power exerted on them but what ariſes from the compounded centrifugal and gravitating Forces; to it the Celeſtial Bodies would appear in the Viſible Horizon at their Riſing and Setting, were it not for the Effects of Refraction, but as the Eye muſt become ſituated above the Surface of the Sea, it ſees the Celeſtial Bodies ſooner at Riſing and later at Setting than it otherwiſe would; likewiſe, at any Altitude above the Horizon, it ſees the Celeſtial Bodies under a greater than the true Elevation by this Quantity; for this Reaſon, it is called the Dip of Horizon.

2. A Table of the Dip of Horizon, is commonly for each Foot of Altitude, oppoſite to which the Dip is ſhewn in Minutes and Seconds, and as the Heighth increaſeth the Dip increaſeth with it; but in all Caſes, the Height will be very ſmall being compared with the Diſtance (or fartheſt Sight of the Sea) eſpecially in ſmall Heights; and therefore, when Accuracy is required, Care ſhould be taken to have the Height correct.

3. If the Eye was ſituated in the Surface of the Sea, the Viſible Horizon would be a Plane and the Eye would be at a Point in it; but, when the Eye is elevated above that Surface, the Dip is in the ſlant Side of a Cone, which has its Vertex for the place of the Eye, and its Altitude for the Elevation above the Surface of the Sea. As that Height is increaſed, ſo the Eye can ſee more Surface of the Sea, but can never ſee Half the Surface (ſuppoſing the whole in a Fluid State) unleſs it be removed to an infinite Height or Diſtance.

4. Whilſt the Eye is at no greater Height than the parts of a Ship will admit, and a Ship is approaching toward Land, the higheſt Parts of the Lands firſt appear coincident with the Line of the Dip of Horizon (at this time the Surface of the Sea near thoſe Lands, is more depreſſed than the Dip of Horizon) next the leſs elevated Parts of the Lands appear, and then the Surface of the Sea near the Lands, in the Line of the Dip of Horizon; when near the Land, Waters near the Coaſt, become again more depreſſed than the Dip of Horizon, and this naturally follows from the Motion of the Eye and of the conical Baſe bounding the Sight.

LXXVII.

Of Ships that are Landlocked, or otherwiſe happen to be where the Horizon of the Sea is inviſible.

1. When Ships are in Sight of Land and are ſailing toward it, the Horizon of the Sea diſappears through the Interpoſition of the Land. The ſame thing happens to Ships when they are in Harbours and ſailing nearly parallel to any Coaſt; at ſuch times, the Altitudes of the Celeſtial Bodies toward the Lands cannot be taken above the Horizon of the Sea, and therefore other Methods muſt be applied.

2. In ſuch Caſes, the uppermoſt Parts of the Lands (if they are even and well defined) or any other Parts, to the Line of the Coaſt itſelf, perpendicularly beneath, may be made uſe of inſtead of the Horizon of the Sea; but, the Elevation thereof above, or Depreſſion thereof below the Horizon of the Sea, muſt be known before it can be applied.

3. The Height of the Eye at any Part of the Ship is known, the Angle made with the Land and

the Visible Horizon may be taken various ways, and therefore the Elevation or Depression of the Part to be used may be taken and allowed for, in Altitudes depending thereon. Thus,

4. When the Height of the Land is but small the floating Mirror or parallel-plane Glass floating on Quicksilver may be applied. Here, the Celestial Body, by double Reflection in Hadley's Sextant, is brought down to the Image thrown up by single Reflection from the Mirror, and the Degrees and Minutes on the Instrument are twice the observed Altitude above the Visible Horizon, to be corrected only for the Refraction.

5. When either the Land or Water's Edge intended to be used, is lower than the Visible Horizon, the Angle between it and some elevated Part may often be taken, and therefrom the Depression known. But, if this cannot be done, the Angle with the Perpendicular may be taken.

6. In all Cases; whenever the opposite Horizon of the Sea can be seen, the Back Observation with Hadley's Octant, will take the Altitude. More generally and correctly, if the Quadrant invented by me for taking Angles quite to One hundred and eighty Degrees be applied, the Back Observation with Hadley's Octant, is not only unnecessary; but, at any time when the opposite Horizons are visible, they may be united and taken either when their angular distance is greater or less than a Semicircle; and thereby, the present state of the Horizontal Refraction, at any time known.

7. This Instrument was mentioned in my Practical Astronomy, and intended to supply the Defects of Hadley's Sextant, in taking Angles greater than One hundred and twenty Degrees. It consists of a Quadrant and two Glasses at the Centre, so placed, that an Angle may be taken quite to a Semicircle, with a large Field of View on the Glasses; and when a Telescope is appled to the Instrument, the apparent Motion of the conjoined Images on the Glasses, is but Half the Motion of the Eye at the Eye-Hole of Hadley's Sextant, to keep the Images in View on the Glasses.

8. Another Property belonging to this Instrument is, its being capable of Verification, either as adjusted or used for an Angle less than a Semicircle, or its Remainder to a whole Circle.

Farther, Hadley's Sextant will not well bear more than a magnifying Eye Telescope of Three times, whilst this will bear six times; and therefore, besides being proper for particular Cases at Sea, it may be applied on Land, in many Cases wherein other Instruments would be insufficient.

LXXVIII.

Of the Methods which Mathematicians and Astronomers have applied, in order to deduce the true Angular Distance of the Sun and Moon's Centres; having the Cotemporary Observations, and the Moon's Horizontal Parallax.

Method the First, for Correction.

1. In order to determine the true angular Distance of the Sun and Moon's Centres, there are taken, the Altitude of the Sun's lower Limb, the Altitude of the Moon's lower or upper Limb, and the angular Distance of their nearest Limbs; their true angular Distance being the Inclination of two right Lines drawn from the Centres of the Sun and Moon to the Earth's Centre.

2. In these Data, the Altitude of the Sun's lower Limb, as taken above the Horizon of the Sea, is affected by a different Quantity of Refraction at different Times and Places; when the Altitude is small, this may amount to several Minutes of a Degree, and enters into the Parts to be computed. The like may be said for the Refraction of the Moon's lower or upper Limb, which is taken at the same Time.

3. When the Moon's lower or upper Limb is taken, the Parallax in Altitude for it, may be nearly the same, or it may not be by many Seconds of a Degree, as it is for the Points of Contact; this also will enter into the Parts to be computed.

4 Whether the lower or upper Limbs are taken, the Altitudes will be (in many Cases) some Minutes of a Degree too little or too great, by the Deviation of Gravity; this also enters into the Computation.

5. The Distance of the Limbs is taken as correctly as possible, and is the only true Datum that enters into the Computation.

6. Having this supposed Co-altitude of the Sun's Centre, this supposed Co-altitude of the Moon's Centre, and the well observed Distance of the Limbs, they are the three Sides of a Spherical Triangle, to which they apply the Common Tables of Refraction (which but seldom agree with the State of the Atmosphere where the Observation was made) the Common Ratio for Parallax in Altitude (which is inadequate, first by reason of the upper or lower Limb of the Moon observed; and Secondly, more so on account of the Deviation of Gravity) and, supposing the Zenith Angle correct, they carry on the Process of the whole Triangle by proportional Parts, for Degrees, Minutes and Seconds, to get the true Distance of Centres.

Method

Method the Second, for Correction.

7. In this Method the Data are commonly as in the firſt Method; but, the Number of Minutes and Seconds additive to or ſubductive from the obſerved Diſtance, is found from the Analyſis of the Problem and its Parts.

8. Of theſe two Methods, the firſt cannot be practiſed without a Variety of Obſtructions which are diſagreeble to the ableſt Calculators, on account of the proportional Parts to be allowed for therein. The Degrees, as well as the Minutes and Seconds in the Data, undergo a Variety of Additions and Subtractions; this is hazardous, as an Error would be fatal for the Reſult. In the Concluſion, the Complication may or it may not admit the Truth, according to the Relations of the Terms in the Data to each other (as hath been before mentioned). Therefore, this Method ſhould be introduced with great Caution, and uſed by none but the moſt expert Computors.

9. In the ſecond Method, the Degrees and (very often) the Minutes of the ſeveral Parts of the Data, are retained nearly in the ſame State as they are given; the Operation is for the Correction only, whether it be additive or ſubductive; here are large Data for ſmall Quæſita (the greateſt Advantages that Calculations can have) and therefore, of all others, this Method deſerves the Preference. Beſides, amidſt the ſeveral Errors that ariſe through not knowing the accurate Refractions, Parallaxes in Altitude, and the Deviation of Gravity; the original Triangle here undergoes no Change through all its Parts, previous to the Operation for Correction. Therefore, this is generally the ſhorteſt and moſt correct Method of Solution.

LXXIX.

Of Calculations for correcting the obſerved Diſtance of Centres of Sun and Moon, or of Moon and Star.

1. There are two different Ways of finding the Corrections to be made for the true Diſtance of Centres. The firſt Way is by finding the Effect of Refraction only on the Obſerved Diſtance; this being found and added to the Obſerved Diſtance, clears it from Refraction. Next, the Effect of Parallax on that Diſtance, is to be found and added to or ſubtracted from that Diſtance; this gives nearly the true Diſtance. Laſtly, when the Diſtance is but a ſmall number of Degrees, or one of the Luminaries is not many Degrees above the Horizon; a ſmall Correction, either additive or ſubductive, is to be applied; and this gives the true Diſtance required.

2. In the ſecond Way, the Difference between the Refraction and Parallax in Altitude, is to be found for each of the Luminaries, the Spheric Angle at each of the Luminaries is alſo to be found; the Angles are to be applied (with that Difference and the Diſtance) as Plane Angles; and thereby the principal Correction for Refraction and Parallax found. The final Correction is alſo to be applied, and this gives the true Diſtance required.

LXXX.

Of Linear Tables; their Conſtruction and Uſe.

1. When a Table conſiſting wholly of Numbers, increaſeth in the Value of its Numbers, deſcending perpendicularly, but decreaſeth in the Value of its Numbers, horizontally; and the tabulated Numbers are for whole Degrees, it will be no ſmall difficulty with ſome Perſons, to find the proportional Parts for intermediate Minutes, and eſpecially when there is Inequality in the Differences, under both of theſe Directions.

2. There are Inſtances in ſuch Tables, wherein the Differences either increaſe or decreaſe by very unequal Differences one of theſe ways, and by very different unequal Differences the other way; and ſo, that Proportionality itſelf will not give the Truth, nor any thing leſs than the Method of firſt or (ſometimes) of ſecond Differences. When ſuch Tables fall into the hands of Perſons who are not good Computors, the intermediate Numbers are generally unſought for and thoſe for the whole Degrees uſed in their ſtead; but ſuch introduce Error.

3. The Linear Tables deſcribed in my former Treatiſe on that Subject; were originally deſigned by me for taking off theſe and many other Difficulties, and have anſwered accordingly.

LXXXI.

Of Linear Table One; for Refraction in Diſtance.

1. This Table is deſigned for taking off the Difficulties (and even Errors) that would ariſe in expreſſing its Contents after a Numerical manner; and at the ſame time for compriſing the whole in a ſmall Compaſs, without rendering it leſs correct or leſs ready for Uſe.

2. This Table is formed of Lines of three kinds, which have their reſpective Numbers ſet to them. Two of theſe Kinds are ſtraight Lines; the third Kind is as near a ſtraight Line as the Subject will admit; and the Increaſe or Decreaſe of Diſtance amongſt theſe Lines, is ſuited for the exact proportioning of the Whole.

3. When the Altitude of Sun or Star, the Altitude of the Moon, and Diſtance of Sun and Moon are given; the leſſer of the two Altitudes is to be found at the Top of the Table, between Four Degrees and Fifty Degrees; and before it, is its ſtraight Line going into the Table. The greater of the two Altitudes is to be found at the ſide of the

Table, between Eight Degrees and Ninety Degrees; and at the Angle of Meeting, amongst the (almost straight) Lines crossing the Table, is the Number (by its Line of the third Kind) to be taken out of the Table. This is called Number *A*.

4. In the Data, there may be odd Minutes over and above the whole Degrees, these naturally carry the Point of Meeting amongst the Lines (at Discretion) a little farther than the whole Degrees, for the two Altitudes, and when the Point of Meeting is found, the Interval of Distance between it and the nearest Line of the third Kind, shews the Number near enough for practical Use. It might be said that, the nearest Line to which the Point of Meeting falls, has its Number, and that may be used with but small Error.

5. The best Observations that have been made at different Places of the Earth, do prove that, at the Altitude of a few Degrees, at Sea, the Refraction may be different from what it is supposed to be (compared with Observations that have been made on Land) and that such Differences, may be the Causes of unexpected Errors, in taking the Longitude by the Lunar Method. There are Proofs that, as high as Eight or Ten Degrees of Altitude, the Refraction has been found (in distant Climes) different from what it has been for the Places the Refraction Tables have been made for. At higher Altitudes these Differences do almost vanish, and therefore the Success of the Lunar Method depends on them.

6. From these Considerations, the Rule for taking out a Number from this Table is thus. If no great Accuracy is wanted, the nearest Line of the third Kind to the point of Meeting, is sufficiently exact. If Accuracy is required, take such Parts of Ten, as the Point of Meeting goes beyond the third Line; add it thereto, and with its prefixed Index Two, it is the Number to be taken out of this Table.

7. It may be objected that, sometimes the two Altitudes are such, that the Table cannot be entered with them, Altitudes that are nearly equal are of this Kind, and others; in such Cases, Table Three is to be applied as directed farther on.

LXXXII.

Of Linear Table Two; for Refraction in Distance.

1. In order to apply this Table, the Distance observed in Degrees (and Parts of Degree, if Accuracy is required) must be known, this is called Number *L*. In this Table, opposite to Number *L* is Number *B*, either additive to or subductive from *A*; this gives the whole Effect of Refraction, and which is always to be added to Number *L*, to get the whole Distance cleared from Refraction.

LXXXIII.

Of Linear Table Three; for Refraction in Distance.

1. It has been before noted that, frequently the two Altitudes are such, that they will not meet within but without Table One. In such Cases, Table Three is to be applied thus.

2. With the observed Distance (called *L*) enter Table Three, and opposite thereto is the Number of Seconds of a Degree for the Effect of Refraction; this turned into Minutes and Seconds, and added to the Observed Distance, gives that Distance cleared from Refraction.

LXXIV.

Of Linear Table Four; for Parallax in Distance.

1. This is a Table to be used at no other time but when the principal Effect of Parallax has been found, and a small Correction called *F* is to be found additive to or subtractive therefrom, to give the true Distance cleared from Refraction and Parallax. This Application belongs to farther on.

2. Previous to the Use of this Table, the Moon's Parallax in Altitude must be found to the nearest Minute of a Degree, this is to be found at the Side of the Table; the Distance in whole Degrees is to be found on the other Side of the Table. With this Parallax in Altitude and the Distance, go into the Table, and amongst the Curve Lines, is the Line of a certain Number of Seconds of a Degree. Also, with the principal Effect of Parallax in Minutes of a Degree, and the Degrees of Distance, enter the same Table in the same manner, and take out the Seconds of a Degree from amongst the Crooked Lines. The Difference of these Seconds is the Correction, either additive or subductive, as the Rules direct. The same peculiar Advantages attend this Table, as attend Linear Table One.

LXXXV.

Of Linear Table Five; for Proportional Parts.

1. In this Table, the Lines marked 1, 1; 2, 2; 3, 3; &c. do each of them contain Ten equal Parts. Therefore, if the Distance of any Two Lines of the Third Kind, in Linear Table One, be applied amongst the Lines in this small Table, that Part of the Table is a Measure for Ten of these Parts; consequently, any Number of those Parts less than Ten, may be thereby readily measured to the nearest Unit, which (as before mentioned) need not be, in general Practice.

LXXXVI.

Of Linear Table Six; for Refraction in Distance.

1. This Table is intended as a Supplement or additional

Table, to be ufed with Table Two, when either of the two Altitudes is lefs than Eight Degrees and not under Four Degrees. It is very improper to apply the Lunar Method of finding the Longitude at Sea, when either of the Luminaries has an Altitude lefs than Four Degrees; on Account of the Changes that happen in the Refraction, between that Altitude and the Horizon, at different times and places where Ships navigate.

2. When the two Altitudes are between Eight and Four Degrees; the Numbers *A* and *B* are to be found by Tables One and Two (as before directed) then, oppofite to the Diftance in Degrees, in this Table Six, is a Number of Seconds, to be added to or fubducted from Number *B* (as this Table directs) to give the true *B*, which is to be added to or fubtracted from *A* (as fhewn by Table Two) this gives the Effect of Refraction, to be reduced and added to the Obferved Diftance as before directed.

LXXXVII.

Of Linear Table Seven; for Refraction in Altitude.

1. In finding the Correction for the true Diftance of Centres of Sun and Moon, or Moon and Star, by working with the Angles at Sun and Moon, or Moon and Star; it is convenient to have the Refraction in Altitude in Seconds of a Degree, in order to be the more readily fubtracted from the Moon's Parallax in Altitude in Seconds of a Degree. This Table is therefore of that Form, and defigned for fhewing the Refraction in Altitude by Infpection, for all Altitudes of the Luminaries that are moft to be depended on, in the Lunar Method of finding the Longitude at Sea.

2. This Table is adapted for fhewing the Mean State of Refraction for the Britifh Channel, in Spring and Autumn; and therefore (in confiderable Altitudes) may be applied at moft parts of the Temperate and Torrid Zones.

3. If, at any place on Land or at Sea, the Refraction near the Horizon fhould be taken, and found different from what it is in this Table; if it be taken lefs, go forward from the Beginning of the Table, if it be more, go backward as many Seconds as the Difference is; and then from the obferved Altitude in the Table, go either forward or backward by the fame Difference of Altitude, and oppofite thereto is the Refraction required.

4. As fuch Differences are greateft near the Horizon and fmall toward the Zenith, in confiderable Altitudes they almoft vanifh, and therefore the Tabular Numbers themfelves may be ufed.

LXXXVIII.

Of Linear Tables Eight, Nine, Ten, Eleven; for Proportionality, in the Lunar Method of finding the Longitude at Sea.

1. Thefe are four Parts of one Table, as appears by Infpection; and are difpofed in this manner for the more readily fhewing the Numbers contained in them at one View.

2. The Logarithms which are now generally ufed by Mathematicians and Aftronomers, for fhortening their Calculations, are firft, Common Logarithms, or the Logarithms of Natural Numbers that increafe in a fucceffive Order from Unity to four or more Places of Figures; fecondly, Logarithmic Sines, Tangents, Secants, and Complements-arithmetical, from the Beginning to the End of the Quadrant, and in fome Cafes to the End of the Semicircle.

3. There is a third Kind of Logarithms, which are very ufeful for fhortening Calculations on particular Subjects, and they are called Logiftic Logarithms (becaufe the Ancients named the Divifions and Subdivifions into Sixty parts Logiftics); thefe are here formed and applied in a manner very different from their Application by any other Perfon; and I name them Proper Logarithms, becaufe they are proper for fhortening the Calculation in the Lunar Method of finding the Longitude at Sea.

4. When it is intended that four Terms fhall be in Direct Proportionality, three of them being given, and the firft and third Terms do each confift of Minutes and Seconds, but the fecond Term is either Unity or any Whole Number, and the Fourth Term is to be of the fame Kind or Denomination as the fecond; in order to take off the trouble of reducing the firft and fecond Terms to Seconds, and a tedious Operation by Multiplication and Divifion, thefe Proper Logarithms being applied, give the Anfwer by a fingle Subtraction.

5. Thefe Logarithms become eafily applicable with other Logarithms, in Corrections for Refraction, Parallax, and others which requires no more than four Places of Figures befide Index, and therefore they are commonly formed of that Number; thus they are in this Table.

6. From the Order of the Numbers in this Table, it appears that, at its Beginning, the Logarithms decreafe very faft at each Second of a Degree; but, toward the End of the Table, they keep nearer to a ftate of Equality. On this account, it is obvious that through a great Part of the Table, every fingle Second of a Degree and Logarithms for them, are unneceffary, and that every third Second of a Degree, with the correfponding Logarithms, are fufficient for practical

 Purpofes,

Purposes, therefore, this is the Order of the Table; nevertheless, where great Accuracy is required, the logarithmic Differences for each three Seconds of a Degree, near the Foot of the Table, may be applied for that purpose. In the Construction of this Table, Advantages superior to these Disadvantages, arise from the last Numbers in Degrees and Minutes at the Foot of this Table.

LXXXIX.

The Description and Application of the Table of Proper Logarithms.

1. The Top of this Table begins with Nothing, and increaseth from one Minute to another till it comes to Three Degrees.

2. The intermediate Seconds, to every Three Seconds of a Degree are in the Side of the Table, going on from nothing to three, six, nine, to sixty Seconds; and near the Bottom of each Column, are the logarithmic Differences, answering to every Three Seconds of a Degree for near the Middle of that Column.

3. On the Outside is another Column of Seconds of a Degree, whose Differences are also Three Seconds of a Degree, and which are the Mediums between the other Three Seconds. These are for taking out the logarithmic Proportional Parts, or the Seconds from the Logarithms, answerable to the nearest Second, by Inspection.

4. The Row of Degrees and Minutes at Bottom, increasing from Nothing to Forty-five Degrees, is for saving the Computor the Trouble of making several Reductions, which otherwise he might be necessitated to make, in the Application of this Table to the Lunar Method, and at the same time the Conclusion hereby becomes more accurate, on account of the Parts that might be lost through Reductions.

Rules for taking out the Proper Logarithms.

5. *1st.* In using this Table, when the Degrees and Minutes are found at the Top of the Table, and the Seconds can be had exactly in the innermost Column at the Side, the Proper Logarithm thereof is at the Angle of Meeting in the Table, with its prefixed leading Figure next above it. *2d.* When the Seconds are found to Half a Second in the Outermost Column, the Medium of two Proper Logarithms at the Angle of Meeting in the Table, is their Logarithm. Hence it follows that, Proper Logarithms to the nearest Second, and the contrary, may be readily taken out of this Table.

6. Since the inner and outer Column of Seconds do exhaust all Numbers in the Table to the nearest Half Second, the carrying of the whole Table to every Second, would have been unnecessary, and have introduced a Repetition of the same Logarithms, through the latter Part of the Table, without any Signature for the exact Second to which they belonged. By this Method of Trisection, such are avoided, and a much readier Use is introduced in their stead.

XC.

Of Linear Table Twelve; for Reduction, either of Hours, Minutes and Seconds of Time, to Seconds of Time; or of Degrees, Minutes and Seconds, to Seconds of a Degree.

1. This Table begins at Nothing and goes on either to Three Hours by every Minute of Time; or otherwise to Three Degrees by every Minute of a Degree; and opposite to either of them is shewn, either the Seconds of Time, or the Seconds of a Degree, by Inspection. Therefore,

2. When Hours, Minutes and Seconds, either of Time or of Degrees, are given to be turned into Seconds; take out the Seconds for the Hours and Minutes, or for the Degrees and Minutes, and add thereto the odd Seconds, this gives the Seconds required.

3. When Seconds either of Time or of Degrees, are given to be turned into either Hours and Minutes, or of Degrees and Minutes, the next less Number of Seconds appear in the Table, and the Hours and Minutes and Degrees and Minutes opposite to them; these Seconds taken from the given Number of Seconds gives overplus or odd Seconds.

XCI.

Of Linear Table Thirteen; for Reductions, from Nothing to Three hundred and sixty Degrees, or Twenty-four Hours of Time.

1. This Table goes on to Sixty Degrees shewing the Hours and Minutes of Time; also, to Sixty Minutes of a Degree, shewing the Minutes and Seconds of Time. Then, it goes on by Ten Degrees, shewing the Hours and Minutes of Time. Hence, from this Table may be taken out Time for Degrees and Minutes of a Degree, when they are given; also, may be taken out Degrees and Minutes of a Degree, and Minutes and Seconds of a Degree, for Time when it is given; and in many Cases almost by Inspection.

XCII.

Of Linear Table Fourteen; for the Moon's Parallax in Altitude, the Altitude and Horizontal Parallax being given.

1. This Table is designed for shewing by Inspection, the Moon's Parallax in Altitude, to less than

than Five Seconds of a Degree, when the Altitude and Horizontal Parallax are known; and in some Cases to greater Exactness.

2. The Degrees of Altitude are to be found at the Top of the Table, and the slant Lines coming from the Degrees of Altitude downward, belong to them. At the Ends of the Table, are the Minutes of the Moon's Horizontal Parallax, and the Horizontal Lines joining their equal Numbers, belong to them.

3. When the Moon's Altitude is given, it is to be found at the Top of the Table, and when the Horizontal Parallax is given, it is to be found at the End; and at their Angle of Meeting is a Point amongst the perpendicular short Lines, between two numbered Lines, one of which is greater and the other less than the Parallax in Altitude required.

4. In this Table the short perpendicular Lines are for every Sixty Seconds of Parallax in Altitude; and the Point of Meeting may be judged of by Inspection to a Twelfth Part thereof, this answers to Five Seconds of a Degree; if greater Accuracy is required, it may be had from this Table, by applying a little more Attention. This Table will be of very great Use on many Occasions in the Practice of Astronomy both on Land and Sea. For,

5. When the Moon's Altitude is taken on the Meridian (or what generally amounts to the same thing, her greatest Altitude is taken) in order to have the Latitude thereby, that Altitude must be cleared both from Refraction and Parallax in Altitude; the Seconds for Refraction are shewn by Inspection in the foregoing Table, and the Seconds for Parallax in Altitude by this Table, and their Difference is the Correction required. Farther,

6. In my Nautic Tables, it is shewn how to take the Moon's Horizontal Parallax, and her Semidiameter is proportionate thereto; so these may be considered as always given, and at times when her Declination is not tabulated, it is expressed by her Angular Distance from the Sun and other Data in the Day-time, and by like Data in the Night, with her angular Distance from the Fixed Stars.

7. As this Table is applicable for clearing the Moon's Parallax in Altitude on the Meridian, so is it for the same purpose out of the Meridian; and as such Data may be often applied for finding the Latitude and the Longitude itself depends thereon, it is of no small Use in Nautical Astronomy. Hence, at any time let the Moon's Altitude to be taken, and cleared from Semidiameter, Dip, Refraction and Parallax in Altitude, this will give the true Altitude of her Centre, prepared for Spherical Computations, with the Sun, Planets or Fixed Stars.

XCIII.

Of the Rules which are to be observed, in computing the true Distance of Centres, either of Sun and Moon, or of Moon and Star; and the Data for that Purpose.

1. When the true Distance of Centres of the Luminaries is to be found by Computation, there must be given three things; first, the Altitude of the Sun's Centre cleared from Dip and Semidiameter, when the Sun is observed; or Altitude of the Star cleared from Dip, when a Star is observed; secondly, the Altitude of the Moon cleared in like manner; and thirdly the observed Distance. If the Method of Computation is by Sun and Moon's Angles, the Luminaries must likewise be cleared from Refraction in Altitude.

XCIV.

Rules for Calculating the Longitude, having Sights of Sun and Moon and the Linear Tables.

Third general Method of finding Longitude at Sea.

1. To the observed Distance of the nearest Limbs, add Half a Degree, the Sum is the Rough Distance of Centres, with which from the Ephemeris take the nearest Hour, and it is the Rough Hour for the Meridian the Ephemeris was made for.

2. With this Rough Hour, take out the Moon's true Semidiameter and her true Horizontal Parallax. Add together, the observed Distance of Limbs, the Sun's true Semidiameter, the Moon's true Semidiameter, and Seconds for Moon's Altitude, the Sum is the observed Distance of Centres.

3. With Sun and Moon's Altitude, take out the Number from Table One; also, with the observed Distance of Centres, its Co-ar. from Logarithms, and add them together, their Sum is a Common Logarithm, whose Natural Number is *A*. With the observed Distance of Centres, take its Number from Table Two, and apply it to the former (either by Addition or Subtraction as directed) this gives the Correction for Refraction. Add this to the observed Distance of Centres, the Sum is the Distance corrected for Refraction.

4. Add together; the Sine of the Distance corrected for Refraction, the Co-secant of Sun's Altitude, and Proper Logarithm of the Moon's Horizontal Parallax; the Sum, (rejecting Tens in the Index) is the Proper Logarithm of Arc the First.

5. Add together; the Tangent of the Distance corrected for Refraction, the Co-secant of Moon's Altitude, and Proper Logarithm of the Moon's Horizontal Parallax; the Sum (rejecting Tens in the Index) is the Proper Logarithm of Arc the Second.

6. Add the Arcs together when the Distance corrected for Refraction exceeds Ninety Degrees, otherwise Subtract one from the other, this gives the Correction for Parallax. When the first Arc

is

is least of the two Arcs, and at the same time the Distance corrected for Refraction is under Ninety Degrees, add the Correction for Parallax to the Distance corrected for Refraction; but otherwise, subtract it from that Distance, this gives the True Distance of Centres nearly, whenever the Distance is great, or Moon's Altitude great.

7. For short Distances or small Moon's Altitude or both; either from Linear Tables or from Nautic Tables, or by Calculation, take the Moon's Parallax in Altitude, to one or two Minutes of a Degree; with this and the Distance take the Seconds from Table Four. Also, with the number of Minutes for Correction of Parallax, and the Degrees of Distance, take the Seconds from the same Table; and the Difference of those Seconds is the final Correction, to be added when the Distance corrected is under Ninety Degrees, but if otherwise to be subtracted; this gives the true Distance of Centres.

8. From amongst the three hourly predicted Distances in the Ephemeris, take two such Distances following each other, so that the true Distance falls between them. Take the Difference between those two three hourly Distances; likewise the Difference between the true Distance and the first of three hourly Distances, and take the Difference between the Proper Logarithms of those two Remainders. Find this Difference in Proper Logarithms, and at Bottom is a Number of Degrees and Minutes, to this add a fourth part of as many Seconds as are in the Side of Proper Logarithms calling them Minutes, and also add the first of the two Times in Degrees; the Sum of these three is the Time past Noon at the Ephemeris's Meridian, in Degrees and Minutes.

9. Take out the Sun's true Polar-distance for the true Hour thus found at the Place the Ephemeris was made for; with this, the Co-latitude and the Sun's Co-altitude find the Solar Time as before directed, and they will be the Degrees and Minutes past Noon for an Afternoon Observation at the Ship, but short of Noon for a Forenoon Observation at the Ship. The first is known by the Sun's increasing and the latter by decreasing in Altitude.

10. In a Forenoon Observation at the Ship, add the Times thus found at the Ship and for the Ephemeris together, their Sum is West Longitude from the Ephemeris's Meridian. In an Afternoon Observation at the Ship, take the Difference of those two Times and the Remainder is East Longitude when the Ship's time is greatest but otherwise West Longitude. When the West Longitude exceeds One hundred and eighty Degrees, subtract it from Three hundred and sixty Degrees, and the Remainder is East Longitude in Degrees and Minutes.

XCV.

Rules for calculating the Longitude having Sights of Moon and Star, and the Linear Tables.

Third general Method of finding Longitude at Sea.

1. To the observed Distance of the Moon's nearest Limb, add a Quarter of a Degree, the Sum is the Rough Distance of Centres. From the observed Distance of the Moon's farthest Limb, subtract a Quarter of a Degree, the Remainder is the Rough Distance of Centres. With either of these, from the Ephemeris, take the nearest Hour, and it is the Rough Hour for the Meridian the Ephemeris was made for.

2. With this Rough Hour, take out the Moon's true Horizontal Parallax. Also, take out the Moon's true Semidiameter and Seconds for Moon's Altitude, and their Sum add to the Distance of nearest Limbs; but their Sum subtract from the Distance of farthest Limbs; this gives the observed Distance of Centres. Then,

3. Proceed exactly the same as was done for Sun and Moon, until the true Distance of Star and Moon are found. With this and the Star and Moon's three hourly Distances, proceed as with Sun and Moon, until the Time is found for the Place the Ephemeris was made for, in Degrees and Minutes past Noon.

4. Take out the Star's true Declination and Right Ascension. Then, proceed with the Co-latitude, the Star's Polar-distance and Star's Co-altitude, as was directed for the Sun; this gives the Star's Equatorial Distance past the Meridian when its Altitude is lessening; and short of the Meridian when it is increasing.

5. Compare together, the Star's Distance from the Meridian, the Star's Right Ascension and the Sun's Right Ascension, and thereby get the Sun's Equatorial Distance from the Meridian, this is the Solar Time at the Ship either past Noon or short of Noon. Compare this (as by Sun and Moon) with the Time at the Place the Ephemeris was made for, this gives the Ship's Longitude.

XCVI.

Rules for calculating the Longitude, having Sights of Sun and Moon, the Linear Tables, and a Watch.

Third general Method of finding Longitude at Sea.

1. In this Method, it is necessary first that the Watch be accurately set either to Solar or Mean Solar Time for the Meridian of the Ship; secondly, that it keeps Time correctly; thirdly, that the

the Ships Change of Longitude after it is ſet to the Time of Obſervation for Longitude be correctly known and allowed for. Then,

2. By the three cotemporary Obſervations the Time at the Place the Ephemeris was made for is had as before for Sun and Moon; and the Sum or Difference between it and the Time ſhewn by the Watch, is the Longitude of the Place the Watch was ſet for.

XCVII.

Rules for calculating the Longitude, having Sights of Moon and Star, the Linear Tables and a Watch.

Third general Method of finding Longitude at Sea.

1. In this Method, it is neceſſary that the Watch be ſet as before mentioned. Then, having the three cotemporary Obſervations the Time for the Ephemeris's Meridian is found as before. The Watch, with proper Allowances (as beforementioned) ſhews the Time for near the Ship. The Sum or Difference of theſe Times is the Longitude.

2. In Night Obſervations, when the Star's Altitude cannot be eaſily nor correctly taken; the Watch is for ſupplying ſuch Defects; but, even then ſhould the Star be not very near the Meridian, a ſmall Error in its Altitude will produce no great one in the Ship's Time inferred therefrom; and if the beſt methods are applied for taking that Altitude, it will be near the Truth, and then there will be leſs dependance on uncertainty that may ariſe in applying the Watch.

XCVIII.

Rules for correcting the Horizontal Parallax.

1. When the Rough Hour is aſſumed, the Moon's Semidiameter and Horizontal Parallax are taken out for that Hour. When the true Hour is found, it may be greater or leſs than the aſſumed; therefore, the true Hour may be put for that firſt aſſumed, and by altering a very few of the ending Figures, the true Diſtance of Centres will be eaſily had, and by proceeding thus, the true Longitude.

2. If the Common Method by help of a Watch be applied, it will be thus. *1ſt.* The Ship's Longitude muſt be known ſomewhat near the Truth; this preſuppoſes the Longitude already known to that accuracy. *2d.* The Ship's Time muſt be known, ſomewhat near; this preſuppoſes ſeveral things found which are ſought. *3d.* The Watch may not be ſet right by the Obſervation. *4th.* It may not go right. *5th.* The Courſe may not be truely known. *6th.* Nor the Variation. *7th.* Nor the Diſtance truly meaſured. All theſe may contribute toward Error through uſe of the Watch; whilſt the cotemporary Obſervations alone, by the foregoing Method, expunge all thoſe Errors, and eaſily give the true Longitude.

XCIX.

Rules for calculating the Longitude, having Sights of Sun and Moon, Common Logarithmic Tables and a Table of Refraction in Altitude.

Third general Method of finding Longitude at Sea.

1. Get the Rough Hour and thereby the Moon's Horizontal Parallax and true Semidiameter as before. Alſo, get the Sun's Altitude cleared from Semidiameter, Dip and Refraction, and the Moon's Altitude cleared from Semidiameter, Dip, Refraction and Parallax in Altitude, reſerving the Difference between the Sun's Refraction and his Parallax in Altitude, and between the Moon's Refraction and her Parallax in Altitude.

2. By the obſerved Co-altitudes and the obſerved Diſtance, get the Angle at the Sun and the Angle at the Moon, each in Degrees and Minutes.

3. As Radius, to the above reſerved Difference for the Sun; ſo is Co-ſine of the Sun's Angle, to a Number of Seconds, for the Sun's Correction. As Radius, to the above reſerved Difference for the Moon; ſo is Coſine of the Moon's Angle, to a Number of Seconds, for the Moon's Correction.

4. When the Sun's Angle is under Ninety Degrees, its Correction is additive; but otherwiſe ſubtractive. When the Moon's Angle is above Ninety Degrees, its Correction is additive; but otherwiſe ſubtractive. Theſe applied to the obſerved Diſtance, clear it from Refraction and Parallax nearly.

5. As Radius, to Sine of Moon's Angle; ſo is the reſerved Difference for the Moon, to a fourth Number in Seconds. Then, As the Diſtance in Seconds, to this fourth Number; ſo is this fourth Number to twice a final Correction in Seconds; whoſe Half is additive when the Diſtance is under Ninety Degrees, but otherwiſe ſubtractive; this gives the true Diſtance of Centres. Theſe two laſt Proportions, are readily worked by Logarithms. Hence,

6. When the Diſtance is not much greater nor leſs than Ninety Degrees, the final Correction is not wanted; but otherwiſe, it will be thus.

Add together, the Log. Sine of the Moon's Altitude and the Common Logarithm of the reſerved Difference in Seconds, double the Sum and ſubtract therefrom the Common Logarithm of the

Diſtance in Seconds; reject all Tens in the Remainder, this is the Common Logarithm of a Number; Half of which is the final Correction, to be added when the Diſtance is under Ninety Degrees, but otherwiſe to be ſubtracted, to give the true Diſtance of Centres.

7. From amongſt the three hourly predicted Diſtances in the Ephemeris, take two ſuch Diſtances following each other, ſo that the true Diſtance falls between them. Take the Difference between thoſe two three hourly Diſtances; likewiſe, the Difference between the true Diſtance and the firſt of the three hourly Diſtances. Then, As the Difference between the two three hourly Diſtances in Seconds, is to the Minutes in Forty-five Degrees; ſo is the Difference in Seconds between the firſt three hourly Diſtance and the true Diſtance, to a number of Minutes, which turned into Degrees, and added to the firſt of the three hourly times in Degrees, gives the Degrees and Minutes paſt Noon at the Place the Ephemeris was made for.

8. Find the Time at the Ship as before for Sun and Moon, and then by the two times (as before) the Longitude.

C.

Rules for calculating the Longitude, having Sights of Moon and Star, common Logarithmic Tables, and a Table of Refraction in Altitude.

Third general Method of finding the Longitude at Sea.

1. Here the Rough Hour, Moon's Horizontal Parallax and true Semidiameter, are taken out as for Moon and Star, in computing by the Linear Tables. Then, the Calculation is as before for Sun and Moon, until the true Diſtance of Centres is found and the Time at the Ephemeris's Place.

2. By the Co-latitude, the Star's Polar-diſtance and the Star's Co-altitude, the Star's equatorial Diſtance from the Meridian is found (as before by Linear Tables) and then the Time at the Ship. The Sum or Difference of the two Times, is the Longitude.

CI.

Rules for calculating the Longitude by Sun and Moon's Angles; having Sights of Sun and Moon, and a Watch accurately ſhewing Solar Time at the Ship.

1. The true Diſtance of Centres is to be found, as by the Sun and Moon's Angles; and then, the Time for the Ephemeris's Place, as before.

2. The Watch is to ſhew the true Solar Time at the Ship, and the Sum or Difference of the two Times is the Longitude.

CII.

Rules for calculating the Longitude, by Star and Moon's Angles; having Sights of Star and Moon, and a Watch accurately ſhewing Solar Time at the Ship.

1. The true Diſtance of Centres is to be found as by the Star and Moon's Angles; and then, the Time for the Ephemeris's Place, as before.

2. The Watch is to ſhew the true Solar Time at the Ship, and the Sum or Difference of the two Times is the Longitude.

CIII.

Of Errors that may happen, in depending on the Watch for ſhewing true Solar Time at the Ship.

1. When a Watch has been carefully regulated, in order to go at Sea, it has been regulated to Equal Time, and generally that is Mean Solar Time. Afterward, at Sea, this is the Time it is ſuppoſed to keep, whether ſhort or long Intervals of Time are meaſured by it; and ſo it goes on, till it is to be applied for ſhewing Solar Time at the Ship.

2. Whilſt the Watch is going at Sea, it may or it may not keep accurately to Mean Solar Time. If it ſhould happen to keep to Equal Time, there is no Standard nor Method at Sea, whereby it can be known. Could the Ship remain a conſiderable while at reſt and Obſervations be repeated with the ſame certainty as on Land, its gaining, loſing or equality might be diſcovered; but, as things are at Sea, the Ship's change of Meridians, the variable Refractions, the Dips, the want of knowing both Latitude and Longitude correctly, for taking out the Declination (and it may be, the want of proper Opportunities for obſerving) are againſt the moſt diligent and able Obſerver. Farther,

3. Whilſt it is at Sea, and is either carefully kept in Cotton, or in the Side-pocket, it may be queſtioned, what Effects the Poſitions and Motion of the Ship may have on it at different times? what Effects from different quantities of Heat, Cold, dry or moiſt Air? what caſual Accidents it may have undergone, and what have not been noted? The Minutes of Time in a Day are Fourteen hundred and forty, ſhould the Watch gain or loſe but One of theſe Minutes in a Day, it is a Quarter of a Degree of Longitude; and that may ariſe from no more than a Fourteen hundred and fortieth part of its Motion, accelerated or retarded in that Interval, through any Accidents.

4. Whether the Watch keeps true Solar Time or not, when it is to be prepared for an Obſerva-

tion,

tion, the Altitude of the Sun can be taken, but the Latitude of the Place may not be correctly known, nor the Sun's Declination (unless the Longitude and Ship's Time be known within certain Limits) these may produce Error.

5. After the Watch is thus set (or rather after an Observation has been made for knowing the Hours, Minutes and Seconds, it is before or after Solar Time at the Ship) there must be Allowances for its gaining or losing (if any); for the Course and Distance sailed; for the Variation, and other Things that incumber the Data to the Time of making the three Cotemporary Observations; and what Errors may arise from such a Complication, is not easily to be determined. After all, the Longitude shewn is for the Meridian where the Watch was either set or examined, and should the Ship run fast and a long Interval, the Distance therefrom may be considerable.

6. This Method of using a Watch to shew Time at the Ship in a Night Observation, has a better appearance, because the Horizon is often obscured in the Night; nevertheless, it may be observed that all the former Errors may attend the use of a Watch in the Night Observation, and besides these (under certain relations of the three cotemporary Observations to each other, and such as frequently happen) the Altitudes must be correct as well as the Distance, or the Error arising from one may be as great as that intended to be corrected by the other; and therefore, the Study of getting correct Altitudes seems (in this Subject) to be of great Consequence.

CIV.

Examples for illustrating the Cases of finding the true Distance of Centres, having the three cotemporary Observations and the Moon's Horizontal Parallax given; taken from the Author's Treatise on the Linear Tables.

1. In these Examples, the Numbers from Linear Table One, and from Co-ar. are to three Figures each beside Index. And,

2. In using the Linear Tables, the Co-secant of Sun's Altitude and Co-secant of Moon's Altitude, are both cleared from Refraction in Altitude. The first and second Arcs, are found by the foregoing Precepts. The other Steps come of course.

3. In computing by Sun and Moon's Angles, or Star and Moon's Angles, *M* is the Angle at the Moon, *S* the Angle at the Sun or Star; *A* the Correction at the Moon's Angle, *B* the Correction at the Sun or Star's Angle. The other Steps come of course.

4. When the two Altitudes are such, that they will not meet within, but without the Table One; then, with the observed Distance of Centres, the whole Effect of Refraction is taken from Table Three.

EXAMPLE I.

" Distance observed	51°	28′	35″"
Star's Altitude	24	48	5
Moon's Altitude	12	30	5
Moon's Hor. Par.	0	56	15

Required the true Distance of Centres?

By Linear Tables.

N° Table I. -	2.136		
Co. ar. of Dist.	0.107		
175″ Log. Sum	2.243		
87 in Table II.	°	′	″
88 Correction	0	1	28
Dist.	51	28	35
D.	51	30	3
1st Arc	0	30	7
2d Arc	0	9	38
C.	0	20	29
D.	51	30	3
E.	51	9	34
F.	0	0	20
P.	51	9	54

By Sun and Moon's Angles.

M.	68°	8′	
S.	93	38	
A.			1134″
B.			8
C.	0	19	2
D.	51	28	35
E.	51	9	33
F.	0	0	20
P.	51	9	53

By different Methods.

1. By the Linear Tables	51°	9′	54″
2. By Sun and Moon's Angles	51	9	53
3. By Mr. Lyons himself	51	9	52
4. By Ditto altered	51	9	51
5. By Mr. Dunthorne himself	51	9	54
6. By Ditto altered	51	9	51

EXAMPLE II.

" Distance observed	90°	21′	13″
Star's Altitude	84	7	6
Moon's Altitude	5	17	8
Moon's Hor. Par.	1	1	48

Required the true Distance of Centres?

By Linear Tables.

N° Table I. -	2.758		
Co. ar. of Dist.	0.000		
573″ Log. Sum	2.758		
0 in Table II.	°	′	″
573 Correction	0	9	33
Dist.	90	21	13
D.	90	30	46
1st Arc	1	1	29
2d Arc	0	0	3
C.	1	1	32
D.	90	30	46
E.	89	29	14
F.	0	0	0
P.	89	29	14

By Sun and Moon's Angles.

M.	3°	0′	
S.	17	20	
A.			3119″
B.			5
C.	0	51	54
D.	90	21	13
E.	89	29	19
F.	0	0	0
P.	89	29	19

By different Methods.

1. By the Linear Tables	89°	29′	14″
2. By Sun and Moon's Angles	89	29	19
3. By Lyons altered	89	29	10
4. By Dunthorne altered	89	29	20
5. By Ditto again altered	89	29	15

EXAMPLE III.

" Distance observed	110°	22′	5″"
Sun's Altitude	45	33	36
Moon's Altitude	19	43	6
Moon's Hor. Par.	0	57	19

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I. -	2.163						
Co. ar. of Dist.	0.028						
155″ Log. Sum	2.191						
41 in Table II.	°	′	″				
196 Correction	0	3	16				
Dist.	110	22	5	M.	19°	38′	
D.	110	25	21	S.	27	56	
1st Arc	0	43	39	A.			2902″
2d Arc	0	7	12	B.			44
C.	0	50	51	C.	0	47	38
D.	110	25	21	D.	110	22	5
E.	109	34	29	E.	109	34	27
F.	0	0	1	F.	0	0	1
P.	109	34	28	P.	109	34	26

By different Methods.

1. By the Linear Tables	109°	34′	28″
2. By Sun and Moon's Angles	109	34	26
3. By Lyons altered	109	34	26

EXAMPLE IV.

" Distance observed	50°	8′	41″"
Star's Altitude	19	18	5
Moon's Altitude	55	56	5
Moon's Hor. Par.	1	0	5

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I. -	2.215						
Co. ar. of Dist.	0.115						
214″ Log. Sum	2.330						
91 in Table II.	°	′	″				
123 Correction	0	2	3				
Dist.	50	8	41	M.	117°	48′	
D.	50	10	44	S.	31	42	
1st Arc	0	25	48	A.			924″
2d Arc	0	41	29	B.			137
C.	0	15	41	C.	0	17	41
D.	50	10	44	D.	50	8	41
E.	50	26	25	E.	50	26	22
F.	0	0	5	F.	0	0	5
P.	50	26	30	P.	50	26	27

By different Methods.

1. By the Linear Tables	50°	26′	30″
2. By Sun and Moon's Angles	50	26	27
3. By Lyons altered	50	26	29
4. By Dunthorne altered	50	26	28

EXAMPLE V.

" Distance observed	28°	14′	39″"
Star's Altitude	20	11	4
Moon's Altitude	18	56	4
Moon's Hor. Par.	0	55	30

Required the true Distance of Centres.

By Linear Tables.				By Sun and Moon's Angles.			
N° Table III.	0°	0′	30″				
Dist.	28	14	39	M.	82°	24′	
D.	28	15	9	S.	87	22	″
1st Arc	0	40	22	A.			395
2d Arc	0	33	26	B.			8
C.	0	6	56	C.	0	6	27
D.	28	15	9	D.	28	14	39
E.	28	8	13	E.	28	8	12
F.	0	0	44	F.	0	0	44
P.	28	8	57	P.	28	8	56

By different Methods.

1. By the Linear Tables	28°	8′	57″
2. By Sun and Moon's Angles	28	8	56
3. By Lyons altered	28	8	57

EXAMPLE VI.

" Distance observed	59°	25′	34″"
Sun's Altitude	59	12	5
Moon's Altitude	27	2	5
Moon's Hor. Par.	0	59	58

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I. -	2.136						
Co. ar. of Dist.	0.065						
159″ Log. Sum	2.201						
65 in Table II.	°	′	″				
94 Correction	0	1	34				
Dist.	59	25	34	M.	35°	4′	
D.	59	27	8	S.	87	42	″
1st Arc	0	59	48	A.			2533
2d Arc	0	16	4	B.			1
C.	0	43	44	C.	0	42	12
D.	59	27	8	D.	59	25	34
E.	58	43	22	E.	58	43	22
F.	0	0	5	F.	0	0	5
P.	58	43	29	P.	58	43	27

By different Methods.

1. By the Linear Tables	58°	43′	29″
2. By Sun and Moon's Angles	58	43	27
3. By Lyons altered	58	43	29

EXAMPLE

EXAMPLE VII.

" Distance observed	43°	35′	42″"
Star's Altitude	11	8	6
Moon's Altitude	9	37	6
Moon's Hor. Par.	0	54	42

Required the true Distance of Centres?

By Linear Tables.			
N° Table I.	2.052		
Co. ar. of Dist.	0.161		
163″ Log. Sum	2.213		
118 in Table II.	° ′ ″	By Sun and Moon's Angles.	
45 Correction	0 0 45		
Dist.	43 35 42	M.	83° 43′
D.	43 36 27	S.	87 50
1st Arc	0 15 24	A.	319″
2d Arc	0 9 32	B.	11
C.	0 5 52	C.	0 5 8
D.	43 36 27	D.	43 35 42
E.	43 30 35	E.	43 30 34
F.	0 0 26	E.	0 0 26
P.	43 31 1	P.	43 31 0

By different Methods.

1. By the Linear Tables 43° 31′ 1″
2. By Sun and Moon's Angles 43 31 0
3. By Astronomer Royal 43 31 2

EXAMPLE VIII.

" Distance observed	29°	24′	46″"
Star's Altitude	49	57	4
Moon's Altitude	64	19	4
Moon's Hor. Par.	0	57	8

Required the true Distance of Centres?

By Linear Tables.		By Sun and Moon's Angles.	
N° Table III.	0° 0′ 31″		
Dist.	29 24 46	M.	42° 4′
D.	29 25 17	S.	95 19
1st Arc	1 44 50	A.	1602″
2d Arc	1 17 34	B.	2
C.	0 27 16	C.	0 26 44
D.	29 25 17	D.	29 24 46
E.	28 58 1	E.	28 58 2
F.	0 0 7	F.	0 0 7
P.	28 58 8	P.	28 58 9

By different Methods.

1. By the Linear Tables 28° 58′ 8″
2. By Sun and Moon's Angles 28 58 9
3. By Witchell's Method 28 58 11

EXAMPLE IX.

" Distance observed	62°	23′	59″"
Star's Altitude	18	42	6
Moon's Altitude	47	42	6
Moon's Hor. Par.	0	57	6

Required the true Distance of Centres?

By Linear Tables.			
N° Table I.	2.188		
Co. ar. of Dist.	0.052		
174″ Log. Sum	2.240		
58 in Table II.	° ′ ″	By Sun and Moon's Angles.	
116 Correction	0 1 56		
Dist.	62 23 59	M.	92° 8′
D.	62 25 55	S.	45 15
1st Arc	0 20 36	A.	84″
2d Arc	0 22 3	B.	116″
C.	0 1 27	C.	0 3 20
D.	62 25 55	D.	62 23 59
E.	62 27 22	E.	62 27 19
F.	0 0 5	F.	0 0 5
P.	62 27 27	P.	62 27 24

By different Methods.

1. By the Linear Tables 62° 27′ 27″
2. By Sun and Moon's Angles 62 27 24
3. By Witchell's Method 62 27 27

EXAMPLE X.

" Distance observed	85°	0′	0″"
Star's Altitude	5	0	5
Moon's Altitude	30	0	5
Moon's Hor. Par.	1	0	0

Required the true Distance of Centres?

By Linear Tables.			
N° Table I.	2.480		
Co. ar. of Dist.	0.002		
303″ Log. Sum	2.482		
9 in Table II.	° ′ ″	By Sun and Moon's Angles.	
294 Correction	0 4 54		
Dist.	85 0 0	M.	87° 6′
D.	85 4 54	S.	60 12
1st Arc	0 5 4	A.	153″
2d Arc	0 2 34	B.	295
C.	0 2 30	C.	0 2 22
D.	85 4 54	D.	85 0 0
E.	85 2 24	E.	85 2 22
F.	0 0 2	F.	0 0 2
P.	85 2 26	P.	85 2 24

By different Methods.

1. By the Linear Tables 85° 2′ 26″
2. By Sun and Moon's Angles 85 2 24
3. By Witchell's Method 85 2 27

EXAMPLE III.

" Distance observed	110°	22′	5″"
Sun's Altitude	45	33	36
Moon's Altitude	19	43	6
Moon's Hor. Par.	0	57	19

Required the true Distance of Centres?

By Linear Tables.

Nº Table I. -	2.163		
Co. ar. of Dist.	0.028		
155″ Log. Sum	2.191		
41 in Table II.	°	′	″
196 Correction	0	3	16
Dist.	110	22	5
D.	110	25	21
1st Arc	0	43	39
2d Arc	0	7	12
C.	0	50	51
D.	110	25	21
E.	109	34	29
F.	0	0	1
P.	109	34	28

By Sun and Moon's Angles.

M.	19° 38′		
S.	27 56		
A.			2902″
B.			44
C.	0	47	38
D.	110	22	5
E.	109	34	27
F.	0	0	1
P.	109	34	26

By different Methods.

1. By the Linear Tables	109°	34′	28″
2. By Sun and Moon's Angles	109	34	26
3. By Lyons altered	109	34	26

EXAMPLE IV.

" Distance observed	50°	8′	41″"
Star's Altitude	19	18	5
Moon's Altitude	55	56	5
Moon's Hor. Par.	1	0	5

Required the true Distance of Centres?

By Linear Tables.

Nº Table I. -	2.215		
Co. ar. of Dist.	0.115		
214″ Log. Sum	2.330		
91 in Table II.	°	′	″
123 Correction	0	2	3
Dist.	50	8	41
D.	50	10	44
1st Arc	0	25	48
2d Arc	0	41	29
C.	0	15	41
D.	50	10	44
E.	50	26	25
F.	0	0	5
P.	50	26	30

By Sun and Moon's Angles.

M.	117° 48′		
S.	31 42		
A.			924″
B.			137
C.	0	17	41
D.	50	8	41
E.	50	26	22
F.	0	0	5
P.	50	26	27

By different Methods.

1. By the Linear Tables	50°	26′	30″
2. By Sun and Moon's Angles	50	26	27
3. By Lyons altered	50	26	29
4. By Dunthorne altered	50	26	28

EXAMPLE V.

" Distance observed	28°	14′	39″"
Star's Altitude	20	11	4
Moon's Altitude	18	56	4
Moon's Hor. Par.	0	55	30

Required the true Distance of Centres.

By Linear Tables.

Nº Table III.	0°	0′	30″
Dist.	28	14	39
D.	28	15	9
1st Arc	0	40	22
2d Arc	0	33	26
C.	0	6	56
D.	28	15	9
E.	28	8	13
F.	0	0	44
P.	28	8	57

By Sun and Moon's Angles.

M.	82° 24′		
S.	87 22		″
A.			395
B.			8
C.	0	6	27
D.	28	14	39
E.	28	8	12
F.	0	0	44
P.	28	8	56

By different Methods.

1. By the Linear Tables	28°	8′	57″
2. By Sun and Moon's Angles	28	8	56
3. By Lyons altered	28	8	57

EXAMPLE VI.

" Distance observed	59°	25′	34″"
Sun's Altitude	59	12	5
Moon's Altitude	27	2	5
Moon's Hor. Par.	0	59	58

Required the true Distance of Centres?

By Linear Tables.

Nº Table I. -	2.136		
Co. ar. of Dist.	0.065		
159″ Log. Sum	2.201		
65 in Table II.	°	′	″
94 Correction	0	1	34
Dist.	59	25	34
D.	59	27	8
1st Arc	0	59	48
2d Arc	0	16	4
C.	0	43	44
D.	59	27	8
E.	58	43	22
F.	0	0	5
P.	58	43	29

By Sun and Moon's Angles.

M.	35° 4′		
S.	87 42		″
A.			2533
B.			1
C.	0	42	12
D.	59	25	34
E.	58	43	22
F.	0	0	5
P.	58	43	27

By different Methods.

1. By the Linear Tables	58°	43′	29″
2. By Sun and Moon's Angles	58	43	27
3. By Lyons altered	58	43	29

EXAMPLE VII.

"Distance observed	43°	35'	42"""
Star's Altitude	11	8	6
Moon's Altitude	9	37	6
Moon's Hor. Par.	0	54	42

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.052						
Co. ar. of Dist.	0.161						
163" Log. Sum	2.213						
118 in Table II.	°	'	"				
45 Correction	0	0	45	M.	83° 43'		
Dist.	43	35	42	S.	87 50		
D.	43	36	27	A.			319"
1st Arc	0	15	24	B.			11
2d Arc	0	9	32	C.	0	5	8
C.	0	5	52	D.	43	35	42
D.	43	36	27	E.	43	30	34
E.	43	30	35	E.	0	0	26
F.	0	0	26	P.	43	31	0
P.	43	31	1				

By different Methods.

1. By the Linear Tables	43°	31'	1"
2. By Sun and Moon's Angles	43	31	0
3. By Astronomer Royal	43	31	2

EXAMPLE VIII.

"Distance observed	29°	24'	46"""
Star's Altitude	49	57	4
Moon's Altitude	64	19	4
Moon's Hor. Par.	0	57	8

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table III.	0°	0'	31"				
Dist.	29	24	46	M.	42° 4'		
D.	29	25	17	S.	95 19		
1st Arc	1	44	50	A.			1602"
2d Arc	1	17	34	B.			2
C.	0	27	16	C.	0	26	44
D.	29	25	17	D.	29	24	46
E.	28	58	1	E.	28	58	2
F.	0	0	7	F.	0	0	7
P.	28	58	8	P.	28	58	9

By different Methods.

1. By the Linear Tables	28°	58'	8"
2. By Sun and Moon's Angles	28	58	9
3. By Witchell's Method	28	58	11

EXAMPLE IX.

"Distance observed	62°	23'	59"""
Star's Altitude	18	42	6
Moon's Altitude	47	42	6
Moon's Hor. Par.	0	57	6

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.188						
Co. ar. of Dist.	0.052						
174" Log. Sum	2.240						
58 in Table II.	°	'	"				
116 Correction	0	1	56	M.	92° 8'		
Dist.	62	23	59	S.	45 15		
D.	62	25	55	A.			84"
1st Arc	0	20	36	B.			116"
2d Arc	0	22	3	C.	0	3	20
C.	0	1	27	D.	62	23	59
D.	62	25	55	E.	62	27	19
E.	62	27	22	F.	0	0	5
F.	0	0	5	P.	62	27	24
P.	62	27	27				

By different Methods.

1. By the Linear Tables	62°	27'	27"
2. By Sun and Moon's Angles	62	27	24
3. By Witchell's Method	62	27	27

EXAMPLE X.

"Distance observed	85°	0'	0"""
Star's Altitude	5	0	5
Moon's Altitude	30	0	5
Moon's Hor. Par.	1	0	0

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.480						
Co. ar. of Dist.	0.002						
303" Log. Sum	2.482						
9 in Table II.	°	'	"				
294 Correction	0	4	54				
Dist.	85	0	0	M.	87° 6'		
D.	85	4	54	S.	60 12		
1st Arc	0	5	4	A.			153"
2d Arc	0	2	34	B.			295
C.	0	2	30	C.	0	2	22
D.	85	4	54	D.	85	0	0
E.	85	2	24	E.	85	2	22
F.	0	0	2	F.	0	0	2
P.	85	2	26	P.	85	2	24

By different Methods.

1. By the Linear Tables	85°	2'	26"
2. By Sun and Moon's Angles	85	2	24
3. By Witchell's Method	85	2	27

EXAMPLE XI.

	°	′	″
" Distance observed	38°	22′	17″"
Star's Altitude	12	27	5
Moon's Altitude	20	9	4
Moon's Hor. Par.	0	57	24

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.095						
Co. ar. of Dist.	0.207						
201″ Log. Sum	2.302						
140 in Table II.	°	′	″				
61 Correction	0	1	1				
Dist.	38	22	17	M.	95°	22′	
D.	38	23	18	S.	73	12	
1st Arc	0	19	49	A.			288″
2d Arc	0	24	54	B.			73
C.	0	5	5	C.	0	6	1
D.	38	23	18	D.	38	22	17
E.	38	28	23	E.	38	28	18
F.	0	0	31	F.	0	0	31
P.	38	28	54	P.	38	28	49

By different Methods.

1. By the Linear Tables	38°	28′	54
2. By Sun and Moon's Angles	38	28	49
3. By Cambridge Tables	38	28	56

EXAMPLE XII.

	°	′	″
" Distance observed	45°	40′	14″"
Star's Altitude	15	54	4
Moon's Altitude	21	26	4
Moon's Hor. Par.	0	56	30

Required the true Distance of Centres.

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.074						
Co. ar. of Dist.	0.146						
166″ Log. Sum	2.220						
109 in Table II.	°	′	″				
57 Correction	0	0	57				
Dist.	45	40	14	M.	88°	25′	
D.	45	41	11	S.	75	22	
1st Arc	0	21	35	A.			83″
2d Arc	0	20	27	B.			50
C.	0	1	28	C.	0	0	33
D.	45	41	11	D.	45	40	14
E.	45	39	43	E.	45	39	41
F.	0	0	23	F.	0	0	23
P.	45	40	6	P.	45	40	4

By different Methods.

1. By the Linear Tables	45°	40′	6″
2. By Sun and Moon's Angles	45	40	4
3. By Cambridge Tables	45	40	7

EXAMPLE XIII.

	°	′	″
" Distance observed	65°	27′	30″"
Star's Altitude	78	18	6
Moon's Altitude	15	21	6
Moon's Hor. Par.	1	0	25

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.349						
Co. ar. of Dist.	0.041						
246″ Log. Sum	2.390						
50 in Table II.	°	′	″				
196 Correction	0	3	16				
Dist.	65	27	30	M.	7°	46′	
D.	65	30	46	S.	140	24	
1st Arc	1	5	1	A.			3259″
2d Arc	0	7	11	B.			9
C.	0	57	50	C.	0	54	28
D.	65	30	46	D.	65	27	30
E.	64	32	56	E.	64	33	2
F.	0	0	1	F.	0	0	1
P.	64	32	57	P.	64	33	3

By different Methods.

1. By the Linear Tables	64°	32′	57″
2. By Sun and Moon's Angles	64	33	3
3. By Cambridge Tables	64	32	59

EXAMPLE XIV.

	°	′	″
" Distance observed	49°	17′	21″"
Star's Altitude	27	15	5
Moon's Altitude	43	20	5
Moon's Hor. Par.	0	58	50

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I. -	2.090						
Co. ar. of Dist.	0.120						
162″ Log. Sum	2.210						
95 in Table II.	°	′	″				
67 Correction	0	1	7				
Dist.	49	17	21	M.	88°	57′	
D.	49	18	28	S.	54	53	
1st Arc	0	35	30	A.			46″
2d Arc	0	34	44	B.			64
C.	0	0	46	C.	0	0	18
D.	49	18	28	D.	49	17	21
E.	49	17	42	E.	49	17	39
F.	0	0	13	F.	0	0	13
P.	49	17	55	P.	49	17	52

By different Methods.

1. By the Linear Tables	49°	17′	55″
2. By Sun and Moon's Angles	49	17	52
3. By Cambridge Tables	49	17	54

EXAMPLE

EXAMPLE XV.

" Distance observed	35°	29′	45″"
Star's Altitude	17	40	5
Moon's Altitude	20	34	5
Moon's Hor. Par.	0	56	24

Required the true Distance of Centres?

By Linear Tables.		By Sun and Moon's Angles.	
N° Table III.	0° 0′ 37″		
Dist.	35 29 45	M.	88° 9′
D.	35 30 22	S.	79 8
1st Arc	0 29 24	A.	97″
2d Arc	0 27 44	B.	33
C.	0 1 40	C.	0 1 4
D.	35 30 22	D.	35 29 45
E.	35 28 42	E.	35 28 41
F.	0 0 34	F.	0 0 34
P.	35 29 16	P.	35 29 15

By different Methods.

1. By the Linear Tables	35° 29′ 16″
2. By Sun and Moon's Angles	35 29 15
3. By Cambridge Tables	35 29 20

EXAMPLE XVI.

" Distance observed	102°	30′	0″"
Star's Altitude	15	25	5
Moon's Altitude	27	30	5
Moon's Hor. Par.	0	57	3

Required the true Distance of Centres?

By Linear Tables.		By Sun and Moon's Angles.	
N° Table I. -	2.112		
Co. ar. of Dist.	0.010		
133″ Log. Sum	2.122		
24 in Table II.	° ′ ″		
157 Correction	0 2 37		
Dist.	102 30 0	M.	65° 0′
D.	102 32 37	S.	56 30
1st Arc	0 15 29	A.	1237″
2d Arc	0 5 51	B.	113
C.	0 21 20	C.	0 18 44
D.	102 32 37	D.	102 30 0
E.	102 11 17	E.	102 11 16
F.	0 0 5	F.	0 0 5
P.	102 11 12	P.	102 11 11

By different Methods.

1. By the Linear Tables	102° 11′ 12″
2. By Sun and Moon's Angles	102 11 11
3. By Lyons himself	102 11 11
4. By 12 Case Method	102 10 54

EXAMPLE XVII.

" Distance observed	33°	15′	0″"
Star's Altitude	48	20	5
Moon's Altitude	64	30	5
Moon's Hor. Par.	0	55	29

Required the true Distance of Centres.

By Linear Tables.		By Sun and Moon's Angles.	
N° Table III.	0° 0′ 35″		
Dist.	33 15 0	M.	91° 58′
D.	33 15 35	S.	40 25
1st Arc	1 15 33	A.	48
2d Arc	1 16 21	B.	38
C.	0 0 48	C.	0 1 26
D.	33 15 35	D.	33 15 0
E.	33 16 23	E.	33 16 26
F.	0 0 6	F.	0 0 6
P.	33 16 29	P.	33 16 32

By different Methods.

1. By the Linear Tables	33° 16′ 29″
2. By Sun and Moon's Angles	33 16 32
3. By Lyons himself	33 16 29

EXAMPLE XVIII.

" Distance observed	56°	17′	44″"
Star's Altitude	53	13	8
Moon's Altitude	64	38	4
Moon's Hor. Par.	1	1	9

Required the true Distance of Centres?

By Linear Tables.		By Sun and Moon's Angles.	
N° Table III.	0° 1′ 1″		
Dist.	56 17 44	M.	32° 16′
D.	56 18 45	S.	21 50
1st Arc	0 58 50	A.	1306″
2d Arc	0 36 49	B.	40
C.	0 22 1	C.	0 21 6
D.	56 18 45	D.	56 17 44
E.	55 56 44	E.	55 56 38
F.	0 0 0	F.	0 0 0
P.	55 56 44	P.	55 56 38

By different Methods.

1. By the Linear Tables	55° 56′ 44″
2. By Sun and Moon's Angles	55 56 38
3. By Lyons himself	55 56 46
4. By 12 Case Method	55 56 50

EXAMPLE

EXAMPLE XIX.

" Distance observed	113°	19′	26″"
Sun's Altitude	5	1	5
Moon's Altitude	29	38	5
Moon's Hor. Par.	0	56	29

Required the true Distance of Centres?

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.478						
Co. ar. of Dist.	0.037						
328″ Log. Sum	2.515						
48 in Table II.	°	′	″				
376 Correction	0	6	16				
Dist.	113	19	26	M.	69°	13′	
D.	113	25	42	S.	54	39	
1st Arc	0	5	13	A.			1010″
2d Arc	0	12	6	B.			339
C.	0	17	19	C.	0	11	11
D.	113	25	42	D.	113	19	26
E.	113	8	23	E.	113	8	15
F.	0	0	9	F.	0	0	9
P.	113	8	14	P.	113	8	6

By different Methods.

1. By the Linear Tables	113°	8′	14″
2. By Sun and Moon's Angles	113	8	6
3. By Witchell's Method	113	8	12

EXAMPLE XX.

" Distance observed	101°	46′	43″"
Sun's Altitude	56	16	0
Moon's Altitude	17	47	0
Moon's Hor. Par.	0	56	40

Required the true Distance of Centres.

By Linear Tables.				By Sun and Moon's Angles.			
N° Table I.	2.240						
Co. ar. of Dist.	0.009						
177″ Log. Sum	2.249						
22 in Table II.	°	′	″				
199 Correction	0	3	19				
Dist.	85	0	0	M.	16°	22′	
D.	101	46	43	S.	35	6	
1st Arc	0	48	9	A.			2936″
2d Arc	0	3	37	B.			28
C.	0	51	46	C.	0	48	28
D.	101	50	2	D.	101	46	43
E.	100	58	16	E.	100	58	15
F.	0	0	1	F.	0	0	1
P.	100	58	15	P.	100	58	14

By different Methods.

1. By the Linear Tables	100°	58′	15″
2. By Sun and Moon's Angles	100	58	14
3. By a foreign Method	100	58	0

REFRACTION AND PARALLAX.

5. In computing by the Linear Tables, the Operations for the Arcs in Example I, are thus.

For the first and second Arcs.

N° D.	51° 30′ 3″	Sine	9.8935
Star's Alt.	24 46 0	Cosec.	10.3779
Hor. Par.	5615	Pr. Log.	0.5051
First Arc	30 7	Pr. Log.	0.7765
N° D.	51° 30′ 3″	Tang.	10.0994
Moon's Alt.	12 26 0	Cosec.	10.6669
Hor. Par.	5615	Pr. Log.	.5051
Second Arc.	9 38	Pr. Log.	1.2714

6. In computing by the Sun and Moon's Angles, those Angles must be found, and then *A* and *B*; in Example I, thus.

For the Moon's Angle.

N° D.	51° 29′	Co. ar.	0.1065
Moon Coalt.	77 30	Co. ar.	0.0104
Star Coalt.	65 12		
Sum	194 11		
Half Sum	97 5	Sine	9.9967
Remainder	31 53	Sine	9.7228
		Sum	19.8364
	34 4	Cosine	9.9182
Moon's Angle	68 8		

For the Star's Angle.

N° D.	51° 29′	Co. ar.	0.1065
Star Coalt.	65 12	Co. ar	.0.0420
Moon Coalt.	77 30		
Sum	194 11		
Half Sum	97 5	Sine	9.9967
Remainder	19 35	Sine	9.5253
		Sum	19.6705
	46 49	Cosine	9.8352
Star's Angle	93 38		

For the Moon's Correction.

Moon's Alt.	12° 30′	Cosine	9.9896
Hor. Par.	3375″	Com. Log.	3.5283
Par. in Alt.	3296	Com. Log.	3.5179
Refraction	252		
	3044	Com. Log.	3.4834
Moon's Angle	68° 8	Cosine	9.5711
N° A.	1134″	Com. Log.	3.0545

For the Star's Correction.

Star's Refraction	123″	Com. Log.	2.0899
Star's Angle	93° 38′	Cosine	8.8019
N° B.	8″	Com. Log.	0.8918
N° C.	1142	or	0° 19′ 2″

7. In these Examples, the near Agreement of the Results, with others from Tables much more voluminous, and Operations far more operose, proves that these Methods are more easy without being less correct, than any other Methods, that have been published.

Of

CV.

Of the predicted Distances of Centres, of Sun and Moon, or Moon and Zodiacal Stars; and their peculiar Positions at different Times and Places.

1. Predictions of the Moon's Distance from the Sun and Zodiacal Stars, are Results of Computations made from the Lunar and other Tables, foretelling what number of Degrees, Minutes and Seconds, those Distances will be, at certain times to come; either at the end of each Hour, two Hours, three Hours, four Hours, six Hours, twelve Hours, or a whole Day.

2. When such Predictions are for each Hour, they become voluminous, but are reduced nearly to an Equality of Differences. When they are for every twelve Hours, the Medium of the Extreme Distances does not agree accurately with the Medium of the Extreme Times; therefore, three hourly Distances are judged proper to be applied, and are used accordingly.

3. In three hourly Predictions, the Moon's apparent Separation from the Zodiacal Stars at the beginning, middle and end of three Hours, may differ a certain number of Seconds of a Degree, on two Accounts; first by reason of the unequable apparent Recession of the Moon from West to East, through the Heavens; secondly, on Account of the Zodiacal Stars being out of the Moon's Path; and (it might be added that) sometime Refraction in Altitude may have a small visible Effect, when either of the Luminaries is not high above the Horizon. The first of these becomes of no great consequence in three hourly Distances, and the latter may be almost annihilated by proper Altitudes; but Inequalities arising from the second Cause, should be carefully provided for in the Construction of the Predictions.

4. As the Moon's Nodes are continually in Motion, the Position of her Path amongst the Fixed Stars in the Zodiac continually varying; this makes her Passage near the same Fixed Stars near the Ecliptic Line itself always near the Path of the Moon's Recession; but the Stars without the Zodiac, produce much greater Inequalities of angular Distances in equal Intervals, when ever the Moon is at no great Distance from them.

5. The Zodiacal Stars which generally preserve the greatest Equalities in the Moon's three hourly Distances, are, Regulus and Spica; next to these are the Stars in Pegasus, but the bright Star Alpha Aquilæ, produces considerable Inequality by this Property, when the Moon is toward the End of the Zodiac. The Stars in Capricorn, seem too small for general Use in the Lunar Method; and the Head of Aries is more difficult to be found than Aldebaran.

6. In the Torrid Zone, the Moon and some one or other of the Zodiacal Stars, frequently are near the same Vertical Circle, such are favourable Opportunities for observing and calculating in the easiest manner. In such Cases, the Dip of Horizon, the Refraction in Altitude, the Semidiameters, and the Parallaxes in Altitude, reduce the observed to the true Distance of Centres. All these come out from the Tables by Inspection, and their Addition or Subtraction is easy; therefore, such Observations should not be neglected. Likewise,

7. In the Torrid Zone, whilst the Moon is either past or short of her Change but a few Days; when her Cusps (or Corners) appear nearly horizontal, and at no great Elevation, the Twilight will be perpendicularly beneath, and the Zodiacal Stars above the Moon; here will be no want of a Visible Horizon, in the Lunar Method by the Zodiacal Stars; and after Sun Rising for the Morning, or before Sun Setting for the Afternoon, Opportunities for verifying, frequently offer by Sun and Moon.

8. In observing the Luminaries near the same Vertical Circle, whether of Sun and Moon, or of Moon and Star; two Observers are all that are wanted; for, in the Day-time, if one person takes the greatest Altitude and another takes the Distance of the Limbs, their Difference (allowing the two Semidiameters) is the Altitude of the lowest In like manner by Night, if one Person takes the greatest Altitude and another the Distance, their Difference (allowing the Moon's Semidiameter) gives the lowest Altitude.

CVI.

Of the Zodiacal Stars and others, which have their Distauces from the Moon predicted for every three Hours throughout the Year.

These Stars have been already treated of in section 24; neverthelss it may be proper to be more particular concerning them here.

1. Alpha Arietis, in the Head of Aries. This Star is so small and otherwise so arranged, that it cannot be readily found and applied by the generality of persons; besides, it is without the Zodiac.

2. Aldebaran, the Southern Eye of Taurus. This Star is easily to be distinguished, by its largeness, colour and position to other Stars. It is within the Zodiac and south of the Ecliptic Line.

3. Pollux, in the Head of the southern Twin. This Star is easily distinguished by its position to Castor and other Stars. It is near the north Bound of the Zodiac.

4. Regulus, the Lion's Heart. This Star is not difficult to be diſtinguiſhed; it is a little northward of the Ecliptic Line, and eaſily to be applied at many places in the Torrid and Temperate Zones.

5. Spica, the Virgin's Spike. This Star, although ſituated at ſome Diſtance from others, is eaſily diſtinguiſhed by its white ſparkling Luſtre. It is a little ſouth of the Ecliptic Line, and applicacable at many places in the Torrid and ſouth Temperate Zones.

6. Antares, the Scorpion's Heart. This Star is eaſily diſtinguiſhed, by its largeneſs and ſituation to other Stars. It is within the Zodiac, ſouth of the Ecliptic Line, and eaſily applicable at many places in the Torrid and ſouth Temperate Zones.

7. Alpha and Beta in Capricorn, are twe Stars within the Zodiac, north of the Ecliptic Line, but not ſo eaſily to be diſtinguiſhed and applied as thoſe beforementioned. On this account.

8. Alpha Aquilæ, the brighteſt in the Eagle, is a Star eaſily to be diſtinguiſhed, by its white Colour and ſituation to other Stars. This Star is many Degrees north of the Ecliptic aud the Zodiac.

9. Alpha and Beta in Pegaſus the Flying Horſe, are not eaſily to be diſtinguiſhed and applied, at many times and places.

10. Fomalhaut in the ſouthern Fiſh, is eaſily to be diſtinguiſhed by its magnitude, and applicable at many places in the Torrid and ſouth Temperate Zones.

11. The foregoing are the Ten Zodiacal Stars, now commonly applied in the Lunar Method of finding the Longitude at Sea; theſe, with the Sun and Moon make up Twelve Luminaries; amongſt which, the eaſieſt to be diſtinguiſhed and applied are; the Sun and Moon at all places where Ships frequently ſail; Aldebaran, Pollux, Regulus and Alpha Aquilæ, are readily applicable throughout the Torrid and North Temperate Zones; Spica, Antares, and Fomalhaut, throughout the Torrid and South Temperate Zones.

CVII.

Of Ephemerides, and the Tables commonly contained in them.

1. An Ephemeris contains a courſe of Predictions of ſuch things as will happen from one Day to another, concerning the Situations, Poſitions and Places of the Celeſtial Bodies, at certain and determinate times. Such Predictions are generally continued throughout the Year. An Aſtronomical Ephemeris, contains things relative to Aſtronomy; a Nautical Ephemeris, things relative to Navigation.

2. An Aſtronomical Ephemeris contains Predictions relative to Heliocentric and Geocentric Aſtronomy; the Motions, Poſitions, Diſtances, and Inequalities, ariſing from the Primary Planets in their Motions round the Sun, and thoſe of the Secondary Planets in their Motions round the Primary Planets. Likewiſe, it contains the Reductions of the Heliocentric Phænomena, for certain and determinate times.

3. A Nautical Ephemeris, contains things that are applicable for determining the true Situations of Places on Land, and of a Ship at Sea. This requires no inconſiderable part of Aſtronomy in its greateſt Extent, beſide many other things relative to Navigation.

4. An Ephemeris is made for ſome particular Meridian, and cannot be correctly uſed for any other Meridian, without making true allowance for their Difference of Longitude. Therefore

5. In an Ephemeris, when a Prediction is oppoſite to a Day of the Month, the Prediction is for the Beginning of that Day; and this is the Noon of the Civil Day, or twelve Hours later than the Civil Account.

6. In an Ephemeris, when Hours, Minutes and Seconds, expreſs a Prediction, they are for true Solar Time paſt the aſtronomical Beginning or civil Noon of that Day; this goes on to Twenty-four Hours, the Beginning of next Day, at Noon. Hence,

7. In every Ephemeris, two things are to be attended to; firſt, the Meridian of the Place for which the Ephemeris is made; ſecondly, the Time of the Prediction. Thus, the Engliſh Aſtronomical Ephemeris, is for the Meridian of the Royal Obſervatory at Greenwich; the French Connoiſſance des Temps, is for the Meridian of the Royal Obſervatory at Paris. Other Works of this kind, are adapted for the Meridians of other Places, except otherwiſe expreſſed.

8. In thoſe Works, the true Diſtance of Sun and Moon; alſo, of the Moon and Zodiacal Stars, are expreſſed for every Three Hours throughout the Year; and in both of theſe for the Meridian of Greenwich Obſervatory.

CVIII.

Having the true Diſtance of Centres of either Sun and Moon, or Moon and Zodiacal Star; to find the Time at the Ephemeris's Meridian.

1. The Time at the Ephemeris's Meridian to be found, will always be paſt Noon; and becauſe the Longitude muſt be in Degrees and Minutes, it

will be readiest to avoid Reduction to Time, and proceed to the Longitude, wholly by Degrees and Minutes. Therefore,

2. *1st.* From amongst the three hourly Distances of Sun and Moon or Moon and Zodiacal Stars, take two such Distances following each other, so that the true Distance of Centres is greater than one and less than the other. *2d.* Take their Difference and call it the first Remainder. *3d.* Take the Difference between the first of the three hourly Distances and the true Distance of Centres, and call it the second Remainder. *4th.* In the Table of Proper Logarithms, look out the Proper Logarithms of these two Remainders, take their Difference and find it in Proper Logarithms. *5th.* At Bottom of the same Column of Proper Logarithms is a number of Degrees and Minutes to be taken out; also, at the Side is a number of Seconds and a fourth part of these are to be called Minutes to be added with the former Degrees; lastly, the first of the Hours taken out are to be turned into Degrees (by allowing fifteen Degrees to an Hour) and the Sum of these three, is the Time past Noon at the Ephemeris's Place, in Degrees and Minutes.

3. If the Time at the Ephemeris's Place be required in Hours, Minutes and Seconds, the Degrees at Top of the Table may be called Hours, and the Minutes and Seconds will be Time; these added to the first Hours gives the Time at the Ephemeris's Place. This introduces Reductions, and therefore is improper to be attended to in the present Method.

4. Otherwise, the Time at the Ephemeris's Place may be found thus. Turn the Difference between the two three-hourly Distances into Seconds of a Degree; and turn the Difference between the first of the two three-hourly Distances and the true Distance, into Seconds of a Degree. To the Common Logarithm of the latter, add 3.4314, and from the Sum subtract the Common Logarithm of the former, the Remainder is the Common Logarithms of a Number of Minutes, the Degrees of which added to the first Distance in Degrees, is the Ephemeris's Time in Degrees and Minutes. Here, 3.4314 is the Logarithm of the Minutes of a Degree in three Hours of Time.

CIX.

Examples for illustrating the Method of finding the Time past Noon, in Degress and Minutes, having the true Distance of Centres, the two three hourly Distances it falls between, and the Proper Logarithms; partly taken from the Author's Treatise on the Linear Tables.

EXAMPLE I.

First Hours	3 Dist.	73°	1′	27″
Second Hours	6 Dist.	74	28	50
First Remainder		1	27	23
First Hours	3 Dist.	73	1	27
True Distance	Dist.	74	11	58
Second Remainder		1	10	31
0.3159 Pr. Log. of 1st Remainder.				
0.4070 Pr. Log. of 2d Remainder.				
0.0931 Pr. Log. gives		36	19	
First Hours		45	0	
Ephemeris Time past Noon		81	19	

EXAMPLE II.

First Distances	3 Dist.	108°	5′	58″
Second Hours	6 Dist.	109	37	16
First Remainder		1	31	18
First Hours	3 Dist.	108	5	58
True Distance	Dist.	109	34	26
Second Remainder		1	28	28
0.2948 Pr. Log. of 1st Remainder.				
0.3085 Pr. Log. of 2d Remainder.				
0.0137 Pr. Log. gives		43	36	
First Hours		45	0	
Ephemeris Time past Noon		88	36	

EXAMPLE III.

First Hours	6 Dist.	93°	57′	36″
Second Hours	9 Dist.	95	32	11
First Remainder		1	34	35
First Hours	6 Dist.	93	57	36
True Distance	Dist.	95	18	6
Second Remainder		1	20	30
0.2795 Pr. Log. of 1st Remainder.				
0.3495 Pr. Log. of 2d Remainder.				
0.0700 Pr. Log. gives		38	18	
First Hours		90	0	
Ephemeris Time past Noon		128	18	

EXAMPLE IV.

First Hours	6 Dist.	62°	28′	43″
Second Hours	9 Dist.	60	59	34
First Remainder		1	29	9
First Hours	6 Dist.	62	28	43
True Distance	Dist.	62	22	51
Second Remainder		0	5	52
0.3052 Pr. Log. of 1st Remainder.				
1.4870 Pr. Log. of 2d Remainder.				
1.1818 Pr. Log. gives		2	$57\frac{1}{2}$	
First Hours		90	0	
Ephemeris Time past Noon		92	$57\frac{1}{2}$	

EXAMPLE V.

First Hours	9 Dist.	116°	33′	55″
Second Hours	12 Dist.	115	5	5
First Remainder		1	28	50
First Hours	9 Dist.	116	33	55
True Distance	Dist.	116	3	26
Second Remainder		0	30	29
0.3067 Pr. Log. of 1st Remainder.				
0.7712 Pr. Log. of 2d Remainder.				
0.4645 Pr. Log. gives		15	26	
First Hours		135	0	
Ephemeris Time past Noon		150	26	

EXAMPLE VI.

First Hours	0 Dist.	48°	2′	33″
Second Hours	3 Dist.	46	33	44
First Remainder		1	28	49
First Hours	0 Dist.	48	2	33
True Distance	Dist.	47	1	7
Second Remainder		1	1	26
0.3068 Pr. Log. of 1st Remainder.				
0.4669 Pr. Log. of 2d Remainder.				
0.1601 Pr. Log. gives		31	7½	
First Hours		0	0	
Ephemeris Time past Noon		31	7½	

EXAMPLE VII.

First Hours	15 Dist.	120°	54′	8″
Second Hours	18 Dist.	119	13	14
First Remainder		1	40	54
First Hours	15 Dist.	120	54	8
True Distance	Dist.	119	50	24
Second Remainder		1	3	44
0.2514 Pr. Log. of 1st Remainder.				
0.4509 Pr. Log. of 2d Remainder.				
0.1995 Pr. Log. gives		28	25½	
First Hours		225	0	
Ephemeris Time past Noon		253	25½	

EXAMPLE VIII.

First Hours	21 Dist.	116°	33′	55″
Second Hours	24 Dist.	115	5	5
First Remainder		1	28	50
First Hours	21 Dist.	116	33	55
True Distance	Dist.	116	3	20
Second Remainder		0	30	35
0.3067 Pr. Log. of 1st Remainder.				
0.7698 Pr. Log. of 2d Remainder.				
0.4631 Pr. Log. gives		15	29½	
First Hours		315	0	
Ephemeris Time past Noon		330	29½	

CX.

Examples for illustrating the Method of finding the Time past Noon in Degrees and Minutes, having the true Distance of Centres, the two three-hourly Distances and the Linear Tables and Common Logarithms.

EXAMPLE I.

First Hours	3 Dist.	73°	1′	27″
Second Hours	6 Dist.	74	28	50
First Remainder		1	27	23
First Hours	3 Dist.	73	1	27
True Distance	Dist.	74	11	58
Second Remainder		1	10	31
2d Remainder 4231″	Com. Log.	3.6264		
Constant Logarithm, add.		3.4314		
	Sum	7.0578		
1st Remainder 5243″	Com. Log.	3.7196		
36° 19′ or 2179′	Com. Log.	3.3382		
45 0 First Hours.				
81 19 Ephemeris Time past Noon.				

EXAMPLE II.

First Hours	21 Dist.	116°	33′	55″
Second Hours	24 Dist.	115	5	5
First Remainder		1	28	50
First Hours	21 Dist.	116	33	55
True Distance	Dist.	116	3	20
Second Remainder		0	30	35
2d Remainder 1835″	Com. Log.	3.2636		
Constant Logarithm, add.		3.4314		
	Sum	6.6950		
1st Remainder 5330″	Com. Log.	3.7267		
15° 29½ or 929½	Com. Log.	2.9683		
315 0 First Hours.				
330 29½ Ephemeris Time past Noon.				

EXAMPLE III.

First Hours	0 Dist.	111°	53′	13″
Second Hours	3 Dist.	110	15	34
First Remainder		1	37	39
First Hours	0 Dist.	111	53	13
True Distance	Dist.	110	32	7
Second Remainder		1	21	6
2d Remainder 4866″	Com. Log.	3.6872		
Constant Logarithm, add		3.4314		
	Sum	7.1186		
1st Remainder 5859″	Com. Log.	3.7678		
37° 22′ or 2242′	Com. Log.	3.3508		
0 0 First Hours.				
37 22 Ephemeris Time past Noon.				

CXI.

Having the Latitude of a Ship at Sea, and the three cotemporary Observations, in the Day time; to find the Solar Time, either short of or past Noon.

1. The three cotemporary Observations are; *1st*, the Altitude of the Sun's lower Limb; *2d*, the Altitude of the Moon's lower or upper Limb; *3d*, the Distance of Sun and Moon's nearest Limbs. The Latitude of the Ship is nearly known, from the Ship's Reckoning and the intermediate run since the last Noon.

2. Add Half a Degree to the Distance of Limbs to get the Rough central Distance, with this from Ephemeris (amongst the three-hourly Distances) take out the nearest Hour for the Ephemeris's Place, and by this Hour take out the Sun's Declination.

3. Having the Latitude, Declination and Altitude, proceed as directed in Section 54, to find the Time at the Ship.

EXAMPLE I.

Co-latitude N.	44°	50′	Co-ar.	0.151782
Polar-distance N.	79	58	Co-ar.	0.006693
Co-altitude	42	50		
Sum	167	38		
Half Sum	83	49	Sine	9.997466
Remainder	40	59	Sine	9.816798
			Sum	19.972739
	14	17	Cosine	9.986369
Solar Time	28	34	short of Noon.	

4. In all Observations that are taken at Sea and on Land, as single Altitudes on either Side of the Meridian, they are for the Spheroidical Earth, which strictly speaking should be for the spherical Figure of the Earth; and therefore, to be correct, the former should be reduced to the latter, thus in this Example.

Lat. N. to Spheroid	45°	10′	Co-lat.	44°	50′
Altitude to Spheroid	47	10	Co-alt.	42	50
Declination N.	10	2	Pol. dist.	79	58

Co-lat.	44°	50′	Co-ar.	0.1518
Co-alt.	42	50	Co-ar.	0.1676
Pol. dist.	79	58		
Sum	167	38		
Half Sum	83	49	Sine	9.9975
Remainder	3	51	Sine	8.8270
			Sum	19.1439
	68	5	Cosine	9.5719
Azimuth	136	10	Cosine	9.8581
Deviation		1160″	Com. Log.	3.0644
13′ 57″	or	837″	Com. Log.	2.9225

47° 10 Alt. to Spheriod.

47° 24′ Alt. to Sphere.
45 10 Lat. to Spheroid.
19 Deviation
44 51 Lat. to Sphere.

Co-lat. to Sphere	45°	9′	Co-ar.	0.149381
Polar-distance	79	58	Co-ar.	0.006693
Co-alt. to Sphere	42	36		
Sum	167	43		
Half Sum	83	51	Sine	9.997493
Remainder	41	15	Sine	9.819113
			Sum	19.972680
	14	17½	Cosine	9.986340
Solar Time	28	35	short of Noon.	

EXAMPLE II.

Co-latitude N.	42°	48′	Co-ar.	0.167848
Polar-distance N.	73	56	Co-ar.	0.017304
Co-altitude	49	50		
Sum	166	34	Sine	9.997009
Half Sum	83	17	Sine	9.741316
			Sum	19.923477
	23	41	Cosine	9.961788
Solar Time	47	22	past Noon.	

Or more correctly, by reducing to the Sphere, thus.

Lat. N. to Spheroid	47°	12′	Co-lat.	42°	48′
Altitude to Spheroid	40	10	Co-alt.	49	50
Declination N.	16	4	Pol-dist.	73	56

Co-lat.	42°	48′	Co ar.	0.1678
Co-alt.	49	50	Co-ar.	0.1168
Pol-dist.	73	56		
Sum	166	34		
Half Sum	83	17	Sine	9.9970
Remainder	9	21	Sine	9.2107
			Sum	19.4923
	56	7½	Cosine	9.7461
Azimuth	112	15	Cosine	9.5782
Deviation		1160″	Com. Log.	3.0644
7′ 19″	or	439″	Com. Log.	2.6426

40° 10 Alt. to Spheroid.
40 17 Alt. to Sphere.
47 12 Lat. to Spheroid.
19 Deviation.
46 53 Lat. to Sphere.

Co-lat. to Sphere	43°	7′	Co-ar.	0.165270
Polar-distance	73	56	Co-ar.	0.017304
Co-alt. to Sphere	49	43		
Sum	166	46		
Half Sum	83	23	Sine	9.997098
Remainder	33	40	Sine	9.743792
			Sum	19.923464
	23	42	Cosine	9.961732
Solar Time	47	24	past Noon.	

EXAMPLE III.

Co-latitude N.	85°	0′	Co-ar.	0.001656
Polar-distance N.	66	32	Co-ar.	0.037492
Co-altitude	84	0		
Sum	235	32	Sine	9.946871
Half Sum	117	46	Sine	9.744928
			Sum	19.730947
	42	48½	Cosine	9.865473
Solar Time	85	37	past Noon.	

Or more correctly, by reducing to the Sphere, thus.

Lat. N. to Spheroid	5°	0′	Co-lat.	85°	0′
Altitude to Spheroid	6	0	Co-alt.	84	0
Declination N.	23	28	Pol. dist.	66	32

Co-lat.	85°	0′	Co-ar.	0.0016
Co-alt.	84	0	Co-ar.	0.0024
Pol. dist.	66	32		
Sum	235	32		
Half Sum	117	46	Sine	9.9469
Remainder	51	14	Sine	9.8919
			Sum	19.8428
	33	26	Cosine	9.9214
Azimuth	66	52	Cosine	9.5942
Deviation		203″	Com. Log.	2.3075
	1′ 20″ or	80	Com. Log.	1.9017
	6°	0	Alt. to Spheroid.	
	6	1	Alt. to Sphere.	
	5	0	Lat. to Spheroid.	
		3	Deviation	
	4	57	Lat. to Sphere.	

Co-lat. to Sphere	85°	3′	Co-ar.	0.001620
Polar-distance	66	32	Co-ar.	0.037492
Co-alt. to Sphere	83	59		
Sum	235	34		
Half Sum	117	47	Sine	9.946804
Remainder	33	48	Sine	9.745300
			Sum	19.731225
	42	47	Cosine	9.865612
Solar Time	85	34	past Noon.	

EXAMPLE IV.

Co-latitude N.	74°	40′	Co-ar.	0.015741
Polar-distance N.	64	4	Co-ar.	0.046094
Co-altitude	64	50		
Sum	203	34		
Half Sum	101	47	Sine	9.990750
Remainder	36	57	Sine	9.778960
			Sum	19.831545
	34	32½	Cosine	9.915772
Solar Time	69	5	short of Noon.	

Or more correctly, by reducing to the Sphere, thus.

Lat. N. to Spheroid	15°	20′	Co lat.	74°	40′
Altitude to Spheroid	25	10	Co-alt.	64	50
Declination N.	25	56	Pol-dist.	64	4

Co lat.	74°	40′	Co-ar.	0.0157
Co-alt.	64	50	Co-ar.	0.0433
Pol-dist.	64	4		
Sum	203°	34′		
Half Sum	101	47	Sine	9.9907
Remainder	37	43	Sine	9.7866
			Sum	19.8363
	34	9	Cosine	9.9181
Azimuth	68	18	Cosine	9.5679
Deviation		600″	Com. Log.	2.7781
	3′ 42″ or	222″	Com. Log.	2.3460
	25°	10	Alt. to Spheroid.	
	25	6	Alt. to Sphere.	
	15	20	Lat. to Spheroid.	
		2	Deviation.	
	15	18	Lat. to Sphere.	

Co-lat. to Sphere	74°	42′	Co-ar.	0.015672
Polar-distance	64	4	Co-ar.	0.046094
Co-alt. to Sphere	64	54		
Sum	203	40		
Half Sum	101	50	Sine	9.990671
Remainder	36	56	Sine	9.778792
			Sum	19.831229
	34	34½	Cosine	9.915614
Solar Time	69	9	short of Noon.	

5. In this Problem, whilst the Sun is over either of the horizontal Quadrants that is nearest to the Point under the elevated Pole, the Sun's Altitude taken is too great; whilst the Sun is over either of the other horizontal Quadrants, the Altitude taken is too little; and when the Sun is over the East or West point of the Horizon, the Altitude taken is near the Truth, or what it would be from a Spherical Earth.

6. The Observations are made on the Surface of a spheroidical Earth or Sea, the Results are to be for that of a spherical Earth or Sea. The three Sides of the spherical Triangle from which the answer arises are, the Polar-distance (this is the same in both Cases); the Co-latitude (this is constant in both Cases); the Co-altitude (this is different in the two Cases). The Difference for the Latitude in its Reduction to a spherical Earth is always additive; that for the Altitudes may be additive or subductive, whilst the Polar-distance is invariable; consequently, the Data for the spherical and spheroidical Earth, are different from each other, and they produce different Results, according to the Relation of the Terms to each other, or the Composition of the Parts.

7. Hence also, Altitudes taken of the Celestial Bodies, by Vertical Instruments adjusted perpendicularly by Plumb-lines, both on the Meridian and out of it, are affected by this Property; and therefore, in finding Solar Time either at Sea or on Land, from single Altitudes taken out of the Meridian, these Corrections are necessary for the Truth.

CXII.

CXII.

Having the Ephemeris's Time, and the Ship's Time, in a Day Observation; to determine the Longitude from the Place for which the Ephemeris was made, and likewise the Ship's Longitude from any other Place, whose Longitude is known.

1. The Longitude of the Ship from the Meridian for which the Ephemeris was made, is had by the last Article, Section 94; and by the same Method, the Ship's Longitude is inferred, from any other Meridian, whose Difference of Longitude is known from the Meridian for which the Ephemeris was made.

2. This Determination is applicable, when the Longitude of a Ship at Sea has been taken, and her direct Course and Distance to any designed Place or Port are required. In this Case, the Latitude and Longitude of the Ship, are given; likewise, the Latitude and Longitude of the Place or Port intended to be sailed to, are given; and the direct Course and Distance are found by the Principles of Navigation.

3. In this Method, well-constructed Charts will shew the Bearing or Course to a tolerable Degree of Accuracy, by Inspection, and the Distance may be had with Ease by proper Methods; so that the whole additional Part, after the Longitude is taken, may be easily and almost instantly performed, sufficiently exact for practical Uses.

EXAMPLE I.

Ephemeris Time past Noon	34°	25′
Ship's Time past Noon	21	12
Ship's Longitude West	13	13

EXAMPLE II.

Ephemeris Time past Noon	14°	32′
Ship's Time short of Noon	27	15
Ship's Longitude East	41	47

EXAMPLE III.

Greenwich Time past Noon	25°	34′
Ship's Time short of Noon	74	16
Ship West of Greenwich	99	50

EXAMPLE IV.

Greenwich Time past Noon	32°	34′
Ship's Time past Noon	10	19
Ship West of Greenwich	22	15
Lizard West of Greenwich	5	15
Ship West of Lizard	17	0

EXAMPLE V.

Greenwich Time past Noon	215°	32′
Ship's Time short of Noon	24	18
Ship West of Greenwich	239	50
Ship East of Greenwich	20	10
C. Good Hope East of Greenw.	18	23
Ship East of Cape Good Hope	1	47

CXIII

Having the Latitude of a Ship at Sea, and the Altitude of a Fixed Star out of the Meridian, to find the Solar Time, either short of or past Noon, and the Longitude.

1. The Fixed Stars which have been observed by Astronomers and had their Places ascertained with respect to the Circles and Poles of the Celestial Sphere, have been of seven different Magnitudes; those of the first Magnitude being of the greatest Splendor or Brightness, and the others diminishing to those which can hardly be discerned by the naked Eye.

2. Those of the first Magnitude can easily be observed at Sea on the Glasses of Hadley's Sextant, by the naked Eye, in favourable Weather, and so can those of the second and third, when a small telescopic Eye-piece is applied; but, the smaller Magnitudes are not easily to be perceived, when their Light has either passed through or been reflected from the Glasses of that Instrument.

3. The Constellations are certain Figures which the Ancients feigned, and allotted certain Parts of the celestial Epanse, with the Fixed Stars within those Parts to represent. Hence, most of the Fixed Stars, have their respective Constellations. The Moderns have added New Constellations, some in the northern and others in the southern Hemisphere of the Heavens to include many Stars that had been before unobserved; this has increased the Number of Stars that were formerly tabulated and delineated on Globes and Charts of the Fixed Stars.

4. The principal Fixed Stars of the first, and between the first and second Magnitude, as they come in order of their Right Ascensions, are, *Achernar*, in the River Eridanus; *Aldebaran*, the Eye of the Bull; *Capella*, in the Shoulder of Auriga; *Rigel*, in the western Foot of Orion; *Betelgeuse*, in the eastern Shoulder of Orion; *Canopus*, in the Ship; *Sirius*, in the great Dog; *Castor*, in the Head of the northern Twin; *Procyon*, in the little Dog; *Pollux*, in the Head of the southern Twin; in the *Oar* of the *Ship*; *Regulus*, the Lion's Heart; the Foot of the Cross; The *Virgin's Spike*; The western Foot of the *Centaur*; *Arcturus*, in Bootes the Herdsman; *Antares*, the Scorpion's Heart; *Lyra*, the brightest in the Harp; *Altair*, the brightest in the Eagle; *Fomalhaut*, in the southern Fish. These twenty Stars are so situated throughout the Heavens, and are so well defined, that at most times and places when and where Ships sail, they may (some or others of them) be applied for taking the Latitude at Sea.

5. Of these twenty Fixed Stars, some are more proper to be applied in taking the Latitude at Sea, than

than others; first, on account of the greater quantity of Light; secondly, for their peculiar Situation in the Heavens. Their Right Ascensions are annually progressive, but their Declinations are some of them additive, others subductive.

6. *Betelgeuse*, in the eastern Shoulder of Orion, and *Bellatrix*, in the western Shoulder, are neither of them many Degrees north of the Equator, and the latter alters in Declination no more than a Minute of a Degree in Ten Years. The former alters but a minute in Declination in Forty Years; this may therefore be considered as a constant Star, during many Years, applicable throughout almost all parts of the Torrid and Temperate Zones.

7. *Lyra*, the brightest in the Harp, alters not a Minute of a Degree in Declination in Twenty Years. *Capella*, in the Shoulder of Auriga; *Castor* and *Pollux*, in the Twins; and *Alpha Aquilæ*, the brightest in the Eagle; these alter slowly in Declination, and are easily applicable for all parts of the Torrid, and the North Temperate Zones.

8. *Sirius*, in the great Dog, (the brightest Star in the Heavens) takes fourteen Years to alter a Minute in Declination. *Canopus*, in the Ship, near Forty Years, to alter a Minute in Declination. *Procyon*; *Rigel* and *Antares*, alter slowly; these are applicable in all Parts of the Torrid and south Temperate Zones.

9. When the Sun is Eighteen Degrees below the Horizon, in the Morning, the Twilight begins; in the Evening, with the same Depression, the Twilight ends; and during the Continuance of Twilight, the Point of the Horizon perpendicularly over the Sun, with a certain Extent north and south thereof, is partially illuminated by the Sun's Rays, so that the Horizon is visible.

10. The Moon is often visible in the Day-time, except within two or three Days of her Change (commonly called the New Moon). The Primary Planets and Fixed Stars to those of the seventh Magnitude, require the Sun to be a certain number of Degrees below the Horizon, when they first disappear before Sun Rising, and appear after Sun Setting. Those Degrees of the Sun's Depression are, for

Venus when horned, five Degrees.
Jupiter and Mercury, ten Degrees.
Saturn and Mars, eleven Degrees.
First Magnitudes, twelve Degreee.
Second Magnitudes, thirteen Degrees.
Third Magnitudes, fourteen Degrees.
Fourth Magnitudes, fifteen Degrees.
Fifth Magnitudes, sixteen Degrees.
Sixth Magnitudes, seventeen Degrees.
Seventh Magnitudes, eighteen Degrees.

11. At many other times in the Night time, the Horizon of the Sea is visible under other Directions from North to South; such are proper times for taking the Latitude by Meridian Altitudes of the Moon, the Primary Planets and Fixed Stars. Altitudes in the Night, taken under other Directions, being applied with the Declination and Latitude, give the Time at the Ship. Thus, *1st*.

12. With the Ship's Co-latitude, the Star's Polar-distance (found as for the Sun) and the Star's Co-altitude, find the number of Degrees and Minutes the Star is short of the Meridian, if increasing in Altitude; but past the Meridian, if decreasing in Altitude. *2d*. Take the Difference between the Right Ascension of the Star, and the Right Ascension of the Sun, in Degrees and Minutes; compare this with the Degrees and Minutes the Star is either short of or past the Meridian; and it gives the Solar Time at the Ship, either short of or past Noon. *3d*. Compare this with the Ephemeris's Time (as before directed) and the Sum or Difference is the Longitude.

13. Having the Right Ascension of the Star, the Right Ascension of the Sun, and the equatorial Distance of the Star, either short of or past the Meridian, the Solar Time at the Ship, and the Ship's Longitude, is found thus.

EXAMPLE I.

Right Ascension of Aldebaran	65°	54′
Right Ascension of the Sun	288	8
Sun East of Aldebaran	222	14
Aldebaran past Meridian	36	49
At Ship's place, short of Noon	185	25
At Ephemeris place, past Noon	89	59
Ship's Longitude, West	275	24
Ship's Longitude, East	84	36

EXAMPLE II.

Right Ascension of Antares	244°	5′
Right Ascension of the Sun	331	25
Sun East of Antares	87	20
Antares short of Meridian	61	25
At Ship's place, short of Noon	148	45
At Ephemeris place, past Noon	80	41
Ship's Longitude, West	229	26
Ship's Longitude, East	130	34

EXAMPLE III.

Right Ascension of Spica	198°	29′
Right Ascension of the Sun	105	56
Sun West of Spica	92	33
Spica past the Meridian	28	33
At Ship's place, past Noon	121	6
At Ephemeris place, past Noon	88	5
Ship's Longitude, West	33	1

CXIV.

Of Time at the Ship, deduced from Equal Altitudes of the Sun, at Sea.

1. Seeing that the Meridian Altitude of the Sun is greater than any other Altitude of the Sun (at any Place of the Earth or Sea) on the ſame Day; every Day at any Place, the Sun will have one Altitude in the Morning and another Afternoon, equal to each other, and the Mid-time between thoſe Altitudes will be near Noon.

2. If the Obſerver continues at the ſame Place during the Interval, and the Sun's Declination has no perceptible Change, the Mid-time will be that of the Meridian Tranſit, or Time of Solar Noon; if the Declination alters, the Mid-time will be either before or after Solar Noon.

3. Hence ariſes a Method of ſetting a Watch or Time-keeper (or rather of knowing nearly whether it is before or after Solar Time) by Equal Altitudes; for, if a Ship at Sea be at reſt, when the Sun does not alter in Declination, and two Altitudes are taken, one a little before and the other a little after Noon, and the Times are noted; the Half Sum of the two Times will be exactly Twelve Hours, if the Watch is to Solar Time; and if it be at Mean Solar Time, it will differ therefrom by the Equation of Time. If it be otherwiſe, the Difference will be the Error of the Watch.

EXAMPLE I.

At Sea, June 20th, the Sun.

11h	34′	15″	Alt. Sun's lower Limb	54°	25′
12	30	55	Alt. Sun's lower Limb	54	25
24	5	10	Sum.		
12	2	35	Solar Noon per Watch.		
12	1	9	Solar Noon per Equation.		
0	1	26	Watch before Solar Time.		

EXAMPLE II.

At Sea, December 20th, the Sun.

10h	51′	35″	Alt. Sun's lower Limb	32°	18′
13	3	5	Alt. Sun's lower Limb	32	18
23	54	40	Sum.		
11	57	20	Solar Noon per Watch.		
11	58	27	Solar Noon per Equation.		
0	1	7	Watch after Solar Time.		

4. In like manner the Watch may be examined by Equal Altitudes of the brighteſt Fixed Stars, their Declination during the Interval being invariable.

EXAMPLE III.

At Sea, March 30th, Sirius.

Right Aſcenſion of the Star	6h	35′	44″
Right Aſcenſion of the Sun	0	36	34
Sun Weſt of Star in Right Aſc.	5	59	10
Interval of Equal Altitudes	1	52	14
Half Elapſed Time	0	56	7
Sun Weſt of Star in Right Aſc.	5	59	10
Solar Time at Star's Tranſit	6	55	17
Watch's Time at Mid-time	6	59	57
Watch before Solar Time	0	4	40

In this and other Examples of the ſame kind, the Half Sum of the two Times ſhewn by the Watch, is the Mid-time, or Time of the Star's Tranſit over the Meridian at the Place of Obſervation; or, it is the Watch's Time of the firſt Altitude, added to Half the Elapſed Time.

5. The principal Uſe of this Method is to determine how much a Watch or any other Time-keeper is before or after either Solar or Mean-ſolar Time, at a Time and Place of Obſervation; and there is no doubt of its being practicable on Land with good Succeſs, where Inſtruments with proper Teleſcopes for viewing the Stars, and Clocks undiſturbed by Accidents, can be applied; but, at Sea, the Circumſtances are different. For,

6. Although the Obſerver may take any length of Time to announce the Inſtant when the firſt of the Equal Altitudes is taken, he is confined to an Inſtant for the ſecond of thoſe Altitudes; and if he fails, through Incapacity in himſelf or the Inſtruments uſed, or any other Circumſtance attending the time or place of Obſervation, the Reſult will be accordingly.

7. In Caſes where there is a Change of Declination in the Interval or Elapſed Time, a ſmall Error will ariſe in this Practice; this, in moſt parts of the Torrid Zone may be nearly annihilated, by taking two Equal Altitudes when the Sun is not far from the Meridian; but if the ſame Method be applied, when the Sun's Declination and the Latitude are contrary and widely different from each other, the Sun's ſlow Change in Altitude near Noon, may make the Inſtant of Equal Altitude difficult to be aſcertained, and there may be an Error accordingly.

8. The Deficiency of this Method by Equal Altitudes, and the Difficulty of allowing for the Change of Declination in all Latitudes by Tables adapted for the purpoſe, may be ſuppoſed to have been good reaſons for introducing other Methods in their ſtead; and particularly ſuch as can be practiſed with more Eaſe and Certainty. Amongſt the other Methods that have been propoſed, have

been that by three Altitudes at the Ends of equal Intervals of Time, but this has its Objections, partly for the fame reafons as the former, and if four Altitudes were to be the Data at the end of equal Intervals, the Difficulties would become greater.

9. When the equatorial Diftance of either the Sun or Star from the Meridian, the Sun or Star's Polar-diftance, and the two equal Altitudes have been thus taken, they form the three parts of a Spherical Triangle; in which, the Co-altitude is oppofite to the Angle of Half Elapfed Time; the Polar-diftance is oppofite to the Azimuth from the point under the elevated Pole; and the Co-latitude is oppofite to the Angle at the Sun or Star. The Latitude may therefore be had from thefe Data, by the Solution of a Spherical Triangle; this Method leads directly to the Anfwer, but it requires knowledge of the Circles of the Sphere. It may be eafily computed univerfally from any Altitndes, by the following Methods.

CXV.

Of the Methods which have been propofed by Mathematicians, for finding the Latitude; by having two Altitudes of the Sun, and the Solar Time meafured between the times of Obfervation.

1. When or by whom this Problem was firft invented and propofed is not certainly known, poffibly it was at an earlier time than is generally imagined, fince it has been in Englifh Books of Navigation more than One hundred Years; altho' in thofe former times, the Inftruments ufed in taking Altitudes, and the Time-keepers ufed in meafuring Time, were much lefs perfect than thofe of the prefent Age. Since thofe times, it has been often revived and recommended as of great Ufe in Practical Navigation; notwithftanding this, to the prefent time, it has not gained Credit enough to be brought into univerfal Ufe, but the old Method of taking the Latitude by having Meridian Altitudes of Sun or Stars, has been confidered as more eafy, certain, and almoft wholly to be depended on.

2. As the Problem was firft propofed, and indeed as it now ftands, it is a general one, applicable to the moft interefting Particulars that can be derived from the Phænomena of the apparent diurnal Motion of the Sun, Moon, Planets and Fixed Stars; for, it has no other Data than can be eafily taken at Sea (the Time meafured by a Time-keeper being correct) it is limited to no particular Circumftances; and, its Solution gives the Solar Time of each Obfervation; the Azimuth at each Obfervation, and the Latitude of the Place; except in each of thefe, the Errors in the Obfervations, and in referring the irregular Figure of the Earth to the Heavens.

3. Although the Problem be thus fairly limited, the Operations by the Data, are not fo concife as to become readily applicable, by the generality of Perfons who make long Voyages; for, it has feveral Spherical Triangles, which muft be analized if the Solution is purfued in the moft direct and natural Way, and the parts of each fhould be preferved to the neareft Minute of a Degree; poffibly, in many Cafes, the neareft Half Minute may not be too exact for an Anfwer fufficiently correct. In thefe Operations are four Cafes, namely. (See Plate, Vertical and Polar Spherical Triangles.)

Cafe the Firft, for the Latitude, having two Altitudes and Elapfed Time; by Spherical Triangles.

1ft. When the Latitude and Declination are of the fame Name (that is, both North or both South) then, P will reprefent the elevated Pole, Z the Zenith, ſ the Sun at one Obfervation, S at another Obfervation, if the Altitudes have been taken both before or both after Noon on the fame Day. Confequently, Z P being a part of the Meridian, *a* ſ is one Altitude and ſZ its Co-altitude; *b* S the other Altitude and S Z its Co-altitude, and ſP or S P is the Polar-diftance. Let S ſ be fuppofedly joined by the Arch of a Great Circle of the Sphere, and thereon drop P *d* perpendicular, fo will the Angles made thereby at *d* be Right Angles, and in the two Triangles S P *d* and ſP *d* (each being a Right-angled fpherical Triangle, right-angled at *d*) the Hypothenufe or Polar-diftance is given, alfo the Angles S P *d* and ſP *d*, each of which is Half of the Elapfed Time. Hence, the Arch of a Great Circle S ſ being found, by it and the two Co-altitudes S Z and ſZ, an oblique-angled Triangle S Z ſ is formed. But, in the Right-angled Spherical Triangle, right angled at *d*, the Angle P ſ *d* being found, and in the oblique-angled Spherical Triangle S Z ſ the Angle ſ being found, the one being fubtracted from the other, leaves the Angle P ſ Z; with this remaining Angle and the two Sides P ſ and Z ſ, a Calculation being made, gives the Angle ſP Z, the Angle P Z ſ and the Side P Z; that is, the Hour Angle or equatorial Diftance of the Sun from Noon, the Azimuth when the Sun was at ſ, and the Co-latitude of the Place of Obfervation.

Cafe the Second, for the Latitude, having two Altitudes and Elapfed Time; by Spherical Triangles.

2d. This is when any two Sides of the oblique-angled Spherical Triangle formed above the Horizon,

rizon, are together more than One hundred and eighty Degrees. Thus, if S P and Z P together make more than One hundred and eighty Degrees, the ſupplemental Triangle *p* S *n* is to be ſolved inſtead of the other; then, will be found the Angle at the depreſſed Pole equal to the Angle at the elevated Pole; the Angle at the Nadir equal to the Angle at the Zenith; and the Angle at the Sun, the ſame in both of theſe Triangles. Therefore, to avoid Errors, due care ſhould be taken how thoſe Triangles are formed by the Circles of the Sphere.

Caſe the Third, for the Latitude, having two Altitudes and Elapſed Time; by Spherical Triangles.

3*d*. In the two firſt Caſes, the two Altitudes were ſuppoſed to have been taken whilſt the Sun was on the ſame Side of the Meridian (that is, both before or both after Noon). In this third Caſe, the Latitude and Declination are ſuppoſed both north or both ſouth, and one Altitude taken before, but the other after Noon. Here, the Points S and ſ, are ſuppoſedly joined by the Arch of a Great Circle (as in the former Caſes) and then, the Angles of the right-angled and oblique-angled Spherical Triangles are to be found as before; the Difference of theſe Angles is the included Angle either at ſ or S. Having this Angle, the Polar-diſtance and Co-altitudes, as before; there may be found, the Hour-angles, from Noon, the Azimuths, and the Co-latitude.

Caſe the Fourth, for the Latitude, having two Altitudes and Elapſed Time; by Spherical Triangles

4*th*. This Caſe is when one Obſervation is made before and the other after Noon, on the ſame Day, and the Sum of any two Sides of the Spherical Triangles in the Data, are together more than One hundred and eighty Degrees (for then, the Sum of the ſupplemental Sides of the ſame Triangle below the Horizon, will be leſs than One hundred and eighty Degrees. In this Caſe, the ſupplemental Triangle below the Horizon is to be computed, and the Reſults whether Angles or Sides, to be ſtrictly attended to, according to their Formation by the Circles of the Celeſtial Sphere.

4. Hence it is evident that a direct Method of Solution is no way difficult in either of theſe Caſes, but rather tedious, and (what is ſtill worſe) the relation of the Terms in the Data may be ſuch to each other and the anſwer, that the neglect of a few Seconds of a Degree in an Angle or Side leading to the laſt Reſult, may occaſion a perceptible Error. If therefore ſuch an Error is to be expunged, it muſt be by preſerving the Parts which enter into the Computation, to the neareſt Seconds that can be readily taken out from Tables. In theſe and many other Caſes where Seconds of a Degree are wanted in a Computation, the Logarithmic Tables publiſhed by me, will be found of general and ready Uſe, to any Accuracy that is wanted for nautical Purpoſes.

5. The Difficulties attending the Solution of this Problem, by a direct Method and the original Spherical Triangles, have occaſioned ſeveral other mechanical and mathematical Methods to be introduced in their ſtead. Amongſt theſe, one Method has been propoſed, by help of a large Delineation and the uſual Data, to find the Data under their real and obſerved Circumſtances on the Delineation, and thereby to get the Meridian Altitude. This Method has ſeveral Objections; the Delineation requires greater Extent than can be admitted; alſo, the Perſons who are to uſe it, require greater Dexterity and Ingenuity than moſt Perſons on Sea or Land have; and therefore, ſuch a Method cannot be brought into general Uſe. Another Method has been by means of a braſs Apparatus, depending on central and other Motions, Lines, Diviſions and Subdiviſions; this Method cannot but be attended with conſiderable Expence, as the Workmanſhip alone will require it; and when all is done on this Plan, compare it with the Conſtruction of an Octant or a Sextant, and the many Difficulties to be overcome, in making one of theſe to anſwer without an Error of a Minute of a Degree.

6. The Diſcovery of the Latitude, by having Altitudes taken when the Sun is not on the Meridian, continuing to appear of no ſmall Conſequence in Navigation, about Half a Century ſince Mathematicians began to conſider, by what additional Data the greateſt part of the Difficulties in the Calculation might be removed (it being certain, that both in the Watch Method of finding the Longitude at Sea, when ever Time-keepers could be brought to ſuch Perfection that they might be depended on during a whole Voyage, and likewiſe, in the Lunar Method of finding the Longitude at Sea; the Latitude muſt be known previous to a Calculation for the Time at the Ship) and knowing that from the many and frequent Opportunities of taking the Latitude by Meridian Altitudes of Sun and Stars, the Latitude may thereby be commonly known within certain Limits; they therefore ſubſtituted this Latitude by Account as an additional Datum, and formed Rules from the whole for the true Latitude and the Solar Times of Obſervation at the Ship's Place.

CXVI.

CXVI.

Of the Logarithmic Solar Tables, and Natural Sines, which are commonly used in finding the Latitude; having two Altitudes of the Sun, the Elapsed Time, and the Latitude as kept in the Ship's Reckoning.

1. The Tables by which Calculations in this Method have been commonly made, are Logarithmic Sines, Logarithmic Secants less Radius, Common Logarithms and their Numbers, Natural Sines and their Arches, together with a peculiar Set of Tables called Logarithmic Solar Tables, containing Logarithms of Half Elapsed Time, Logarithms for Middle Time, and Logarithms for Rising to every Half Minute of Time, from 0 to Six Hours. These Logarithmic Solar Tables were constructed by Mr. Cornelis Douwes of Amsterdam, about forty Years since.

2. When the Solar Tables were first printed in this Kingdom, they were without Logarithms of Numbers, Log. Sines, Log. Secants less Radius, and without Natural Sines; the three first were to be taken from the common Books of Navigation, the latter were to be supplied by the Numbers and their Logarithms, making them to answer for Natural and Logarithmic Sines, by altering the Index Figure.

3. In 1771 Mr. N. D. Falk published those Solar Tables, with the additional Logarithms and Natural Sines to five Places of Figures; this was by some persons thought deserving Encouragement. The Nautical Almanac or Astronomical Ephemeris for the same Year 1771 contained those Solar Tables improved to every Ten Seconds of Time, by Capt. John Campbell; these were revised from the Press in 1769 by me.

4. In finding the Latitude by this Method, the two Observations must be made "between Nine in the Morning and Three Afternoon", a great Variety of Particulars are to be remembered to know when the Data are under favourable or unfavourable Circumstances; because, this Method is an Approximation by the first Operation, and that may give the Truth or it may require two or more Operations. Under the most favourable Circumstances, in the first Operation "the Error in the computed Latitude will not be above a fourth Part of the Difference between that by Computation and the Latitude by Account."

5. Whether one, two or more Operations are made by this Method, in each Operation there will be Ten Applications to the Tables; namely, twice for Natural Sines; twice for Secants less Radius; once for the Logarithm of a Number; once for the Logarithm Half the Elapsed Time; once for Hours, Minutes and Seconds in Middle Time; once for the Logarithm of Time from Noon, or Logarithm of Rising; once for a Natural Number, after subtracting Log. Ratio; and once for the Degrees and Minutes of Meridian Altitude.

6. Of these Ten turnings to the Tables, three require proportional Parts, even when the latter improved Tables are used, but more especially when the Half-minute Tables are used; two require Care concerning the Index; these are Obstructions in the Calculation. Beside these, there are together Six Additions and Subtractions, in each first Operation; and in a second or third Operation, only three Logarithms and one Natural Sine, are the same as before, which saves the Calculator so much Time and Care.

7. The Number of these Steps to be taken in every Operation, and their Variety, has been without doubt the Cause of this Method's having been less practised than it otherwise would. If the Solution of a Problem be by a continued Process, without turning many times to Tables, or working for proportional Parts, a few easy Additions and Subtractions will be readily gone through, and the Answer will come out with more Ease and Certainty and in less time, than in taking the same Number of Figures from different Tables; especially when proportional Parts are to be worked for beside, and the Process is after an intricate manner not easily to be remembered.

8. It is farther to be noted that, the greatest Interval of Time between the two Observations is to be no more than Six Hours, in which Interval, when the Sun is near either of the Equinoctial Points, the Sun's Declination alters near Six Minutes of a Degree; and there is no Provision for this or any other Change of Declination in the use of those Solar Tables; nor is there any Method of knowing the Error arising from such or any other Change of Declination.

9. Hence these Inferences. *1st.* Those Tables are applicable, only between Nine in the Morning and Three Afternoon. The limits of this Interval may not be easily determined at Sea; and the want of knowing somewhat near the Time, may occasion several Operations. *2d.* They have no Provision for the Change of the Sun's Declination. This, in usual Cases, may subject the Conclusion to an Error of several Minutes in Latitude. *3d.* The Number of Figures in the logarithmic Solar Tables together with the Natural Sines to be used with them, in the best state they have been yet published, amounts to near Seventy-five thousand; and although these make near four Ninths of the Number of Figures contained in all the Log. Sines, Tangents and Secants to the same number of Places (and these latter are general Tables for Practical Astronomy) yet, those Natural

tural Sines are to but five Places of Figures. In working by this Method and by those Tables, the Natural Sine of the Sun's Meridian Altitude is to be found and it is that which the Latitude depends on. The Natural Sines to five Places of Figures, have their first Figures beginning to repeat at about Eighty-eight Degrees and half of Altitude for each Minute of a Degree; so these Natural Sines fail thus much within about two Degrees and half of the Zenith, and such Altitudes often happen in various Parts of the Torrid Zone. *4th*. The Variety arising in the Application of the Solar Tables, both for the Latitude and in the contrary Problems to which they may be applied, is great, and the Precepts numerous; whilst it is certain that the Intricacies arising from a manifold Variety in Computations, cause Difficulties and Error; the fewer the Precepts are, the better they may be applied; the more numerous they are, the greater need there is of Judgement and Remembrance in the Calculator, and where he is deficient in either of these, and has not the Precepts before him, or (it may be) does not know their meaning, he cannot be expected to be able to work out the Observations. *5th*. Whoever has examined the Methods used by the Antients in constructing Problems by the help of Natural Sines, must know that very easy Constructions were known to them, which would form very difficult Calculations; and that such are become but little better for general Use, since the Invention of Logarithms. The like may be observed concerning many excellent Discoveries made by the Modern Algebra, where the Equation (after all that can be done in reducing it) rests either in an ambiguous, adfected, exponential or some other State, which is improper to be given out for general Use. Any Method of finding the Latitude from those Data, between the same Limits of Interval, as easily and correctly, is as good as that by the Solar Tables. A Method extending to any Numb r of Hours, and at the same time correcting for the Change of Declination, is preferable to the Method by the Solar Tables. A Method which contains all the forementioned Cases, and at the same time solves the Difficulties, without depending on a Watch for measuring the Elapsed Time, is much beyond what can ever be expected from the Logarithmic Solar Tables. Such Methods follow in this Work.

CXVII.

Of Approximations.

1. When the Solution of an astronomical or any other Problem has been pursued, either by the synthetical or analytical Method of Reasoning, and an Expression for the Answer is investigated; such an Expression will sometimes consist of parts easily to be operated and such as give almost the whole Answer, but of such other Parts as may have but little Effect on the Answer yet are with Difficulty to be computed.

2. In such Expressions, the principal and more easily to be computed Parts may be used for the Whole, and the other Parts rejected; in consequence hereof, an Answer brought out by this Contraction and proper Data with the Answer included, will be nearer the Truth than that which was assumed; and if this Answer together with the same proper Data be operated anew, the Answer will come out nearer the Truth than before; and so on, a third, fourth or more times, till the Truth itself is discovered or nearly so.

3. This Method of Computation is called a Method of Approximation, on Account of its approaching nearer and nearer to the Truth at each Operation, and is applicable, when Two Altitudes and Elapsed Time are given, with the Latitude by Account, to find the true Latitude.

4. In these Data, the two Altitudes, Elapsed Time, and Sun's Declination, are real and true. If the assumed Latitude be the true Latitude, the Answer will also be the true Latitude from the Composition of the Data. If the assumed Latitude is greater or less than the Truth, the Answer converges toward the Truth by each Operation, until it is the Truth itself, if sufficiently pursued.

CXVIII.

Of Natural Sines, as they are formed from Logarithmic Sines, Common Logarithms, and their Numbers, in the Author's Treatise of Logarithms.

1. In the Treatise here referred to, as in other Tables, the Index to the Logarithmic Sine of Ninety Degrees is Ten, and it diminishes to Six, at the first Minute of the Quadrant.

2. In the Table of Common Logarithms and their Numbers, the Logarithms without Index are to six Places of Figures; the Natural Numbers belonging to them are to four Places of Figures, and the Indexes are prefixed to the Logarithms, which answer for the Natural Numbers.

3. At the beginning of Nautic Tables, is a Table of Proportional Differences for the Common Logarithms and their Numbers, which is to be used thus. When a Common Logarithm is to be found for six Places of Figures without Index;

find the three first left hand Figures at the Side, the first right-hand Figure of the four at Top, and then go on with the Table of Proportional Differences thus. Find the Difference of Logarithms in Common Logarithms under Diff. in Proportional Differences, the Tens of Numbers with the superadded Number for Units under 10.20 &c. at Top, add these to the Logarithm taken from the Table, and the Sum is that required. The contrary to this finds the Natural Number to six Places of Figures, when the Common Logarithm is given.

4. In making the first Operation for the Latitude, having two Altitudes and the Elapsed Time, with the Latitude by Account, four Figures of Natural Numbers and as many for Logarithms will be sufficient in most Cases; nevertheless, by this Method, six Figures may be had by Inspection when they are wanted, or when it is necessary to take them out at once, without turning a second time to the Tables.

CXIX.

Having two Altitudes of the Sun's Centre, cleared from Semidiameter, Dip and Refraction, and Refraction, and taken between Nine in the Morning and Three Afternoon, the Solar Time measured between the observations, the Sun's Declination, and Latitude somewhat near the truth; to find the true Latitude.

Add together,

1st. Log. Co-sine of half Sum of Altitudes,
2d. Log. Sine of Half Difference of Altitudes,
3d. Co-ar. of Half Elapsed Time,
4th. Secant less Radius of Declination,
5th. Secant less Radius of given Latitude.
In the Sum reject Index Ten; Remainder is Log. Sine of Middle Time from Noon.
Take the Difference between this Middle Time and Half of the Elapsed Time, and take Half of the Remainder.

Then, add together as follows,

6th. Twice the Log. Sine of the Half Remainder.
7th. This constant Logarithm 0.301030,
8th. Log. Co-sine of the Declination,
9th. Log. Co-sine of the given Latitude.
In the Sum, reject Index Thirty, the Remainder is a Common Logarithm, whose Natural Number added to the Natural Sine of the greatest Altitude, gives the Natural Sine of the Meridian Altitude.

1. This Rule is a complete Substitute for the Logarithmic Solar Tables, in their Application for the Latitude having two Altitudes and Elapsed Time. It is taken from the Author's Treatise on Logarithms, and may be rendered more ready for accurate Application and Practice, thus.

2. Take out to six Places of Figures beside Index; *1st.* The Co-sine of the Half Sum of the two Altitudes; *2d.* the Sine of the Half Difference of the two Altitudes; *3d.* the Co-ar. of Half the Elapsed Time; *4th.* the Secant less Radius of the true Declination; add them together and call their Sum the Constant Total to be reserved.

3. In a first Operation, proceed thus. To the Constant Total, add the Secant less Radius, of the Latitude by Account to four Places of Figures besides Index; in the Sum reject Index Ten, the Remainder is the Log. Sine of Middle Time from Noon. *5th.* Take the Difference between this Middle Time and Half of the Elapsed Time and take Half of the Remainder. Then, add together as follows, *6th.* This constant Logarithm 0.301030; *7th.* Twice the Log-sine of the Half Remainder, to four Places beside Index; *8th.* Log. Co-sine of the Declination to six Places beside Index; *9th.* Log. Co-sine of the Latitude, to four Places beside Index. In the Sum reject Index Thirty-six; the Remainder is a Common Logarithm; take out its Natural Number. *10th.* Lessen the Log. sine of greatest Altitude by Six, and with its six Figures beside Index, take its Natural Number (in Common Logarithms) to four Places of Figures, add this to the former Natural Number and the Sum is the Natural Sine of the Meridian Altitude to four Places of Figures; therefore, find its Logarithm in Common Logarithms, and add Six to the Index, the Sum is the Log-sine of the Meridian Altitude.

4. In other words, this Method for a first Operation, may be expressed thus. Take the Sum of the four first Steps to six Figures beside Index, and the Co-sine of the Declination to the same Number, so there will be but once turning to the Tables for them, and four Figures will carry the other Parts through the Process; but, when a second or a third Operation is to be made for the Truth itself, there will be but three other times turning to the Tables, and one Inspection of the Table of Proportional Differences, to have six Figures for the Natural Sine of the Meridian Altitude; which is Ten times as exact as the Solar Tables with their Table of Natural Sines to five Places of Figures.

5. In this Method, the Elapsed Time is commonly supposed such as is measured by a Watch or other Time-keeper keeping very nearly Solar Time during the Interval; for, in such Case, the Time measured by the Watch is very nearly analogous

analagous to the apparent Equatorial Motion. In like manner, a Watch or Time-keeper keeping ſiderial or ſtarry Time may be uſed, when two Altitudes of one and the ſame Fixed Star are obſerved, for the ſame Reaſons; but, if a Fixed Star is obſerved, and the Interval meaſured is ſolar Time, it ſhould be reduced to ſiderial Time, by a proportional Part for the Difference between them.

6. After the ſame manner, the Primary Planets, Saturn, Jupiter, Mars and Venus, may be applied; due allowance being made concerning the Intervals of their apparent Diurnal Revolutions.

EXAMPLE I.

Altitude	28°	54′			
Altitude	20	42			
Sum	49	36			
Diff.	8	12			
½ Sum	24	48	Coſine	9.957979	
½ Diff.	4	6	Sine	8.854291	
½ El. Time	15	0	Co-ar.	0.587004	
Decl. N.	0	0	Sec. leſs R.	0.000000	
Conſtant Total to be reſerved				19.399274	
Lat. Acc. N.	60	0	Sec. leſs R.	0.3010	
Mid. Time	30	6	Sine	9.7003	
½ El. Time	15	0			
Remaind.	15	6		0.301030	
½ Remaind.	7	33	Sine	9.1185	
			Ditto	9.1185	
Decl. N.	0	0	Coſine	10.000000	
Lat. Acc. N.	60	0	Coſine	9.6990	
			Com. Log.	2.2370	N° 172
Great. Alt.	28	54	Sine	3.6842	N° 4833
Merid. Alt.	30	2	Sine	9.6994	Sum 5005
Decl. N.	0	0			
Co-lat.	30	2			
Latitude N.	59	58	by 1ſt Operation.		

Second Operation.

Conſtant Total to be reſerved				19.399274	
Lat. Acc. N.	59°	58	Sec. leſs R.	0.3006	
Mid. Time	30	4	Sine	9.6999	
½ El. Time	15	0			
Remaind.	15	4		0.3010	
½ Remaind.	7	32	Sine	9.1176	
			Ditto	9.1176	
				10.0000	
Lat. Acc. N.	59	58	Coſine	9.6994	
			Com. Log.	2.2356	N° 172
					N° 4833
Merid. Alt.	30	2	Sine	9.6994	Sum 5005
Decl. N.	0	0			
Co-lat.	30	2			
Latitude N.	59	58	by 2d Operation.		

EXAMPLE II.

Altitude	28°	12′			
Altitude	16	49			
Sum	45	1			
Diff.	11	23			
½ Sum	22	30	Coſine	9.965615	
½ Diff.	5	41	Sine	8.995768	
½ El. Time	26	15	Co-ar.	0.354294	
Decl. S.	13	17	Sec. leſs R.	0.011777	
Conſtant Total, to be reſerved				19.327454	
Lat. Acc. N.	47	40	Sec. leſs R.	0.1717	
Mid. Time	18	24	Sine	9.4991	
½ El. Time	26	15			
Remaind.	7	51		0.301030	
½ Remaind.	3	55	Sine	8.8345	
			Ditto	8.8345	
Decl. S.	13	17	Coſine	9.988223	
Lat. Acc. N.	47	40	Coſine	9.8283	
			Com. Log.	1.7865	N° 61
Great. Alt.	28	12	Sine	3.6744	N° 4725
Merid. Alt.	28	36	Sine	9.6800	Sum 4786
Decl. S.	13	17			
Co-lat.	41	53			
Latitude N.	48	7	by 1ſt Operation, which is but half a Minute leſs than the Truth.		

EXAMPLE III.

Altitude	51°	59′			
Altitude	49	9			
Sum	101	8			
Diff.	2	50			
½ Sum	50	34	Coſine	9.802897	
½ Diff.	1	25	Sine	8.393101	
½ El. Time	21	15	Co-ar.	0.440766	
Decl. N.	12	16	Sec. leſs R.	0.010030	
Conſtant Total, to be reſerved				18.646794	
Lat. Acc. N.	47	19	Sec. leſs R.	0.1688	
Mid. Time	3	45	Sine	8.8156	
½ El. Time	21	15			
Remaind.	17	30		0.301030	
½ Remaind.	8	45	Sine	9.1822	
			Ditto	9.1822	
Decl. N.	12	16	Coſine	9.989970	
Lat. Acc. N.	47	19	Coſine	9.8312	
			Com. Log.	2.4866	N° 307
Great. Alt.	51	59	Sine	3.8964	N° 7878
Merid. Alt.	54	56	Sine	9.9130	Sum 8185
Decl. N.	12	16			
Co-lat.	42	40			
Latitude N.	47	20	true Latitude.		

In a ſecond Operation, theſe fifteen Logarithms are reduced to ten; and theſe may be four, five or ſix Figures each.

EXAMPLE

EXAMPLE IV.

Altitude	19°	41′				
Altitude	17	13				
Sum	36	54				
Diff.	2	28				
½ Sum	18	27	Cosine	9.977083		
½ Diff.	1	14	Sine	8.332924		
½ El. Time	7	30	Co ar.	0.884302		
Dec. S.	20	0	Sec. less R.	0.027014		
Constant Total to be reserved				19 221323		
Lat. Acc. N.	50	40	Sec. less R.	0.1980		
Mid. Time	15	14	Sine	9.4193		
½ El. Time	7	30				
Remaind.	7	44		0.301030		
½ Remaind.	3	52	Sine	8.8289		
			Ditto	8.8289		
Decl. S.	20	0	Cosine	9.972986		
Lat. Acc. N.	50	40	Cosine	9.8020		
			Com. Log.	1.7338	N°	54
Great. Alt.	19	41	Sine	3.5274	N°	3368
Merid. Alt.	20	1	Sine	9.5343	Sum	3422
Decl. S.	20	0				
Co-lat.	40	1				
Latitude N.	49	59	by 1st Operation, which is but a Minute less than the Truth.			

EXAMPLE V.

Altitude	21°	55′				
Altitude	17	40				
Sum	39	35				
Diff.	4	15				
½ Sum	19	47	Cosine	9.973580		
½ Diff.	2	7	Sine	8 567431		
½ El. Time	22	20	Co-ar.	0.420223		
Decl. S.	19	30	Sec. less R.	0.025653		
Constant Total, to be reserved				18.986887		
Lat. Acc. N.	47	34	Sec. less R.	0.1709		
Mid. Time	8	16	Sine	9.1577		
½ El. Time	22	20				
Remaind.	14	4		0.301030		
½ Remaind.	7	2	Sine	9.0879		
			Ditto	9.0879		
Decl. S.	19	30	Cosine	9.974347		
Lat. Acc. N.	47	34	Cosine	9.8291		
			Com. Log.	2.2802	N°	196
Great. Alt.	21	55	Sine	3.5720	N°	3733
Merid. Alt.	23	8	Sine	9.5943	Sum	3829
Decl. S.	19	30				
Co-lat.	42	38				

Latitude N. 47 22 by 1st Operation; which is but a Minute and half less than the Truth. Hence, when the Latitude by Account is nearly true; the seven first Logarithms being operated and repeated, shew the Error of the Watch.

EXAMPLE VI.

Altitude	20°	24′				
Altitude	13	50				
Sum	34	14				
Diff.	6	34				
½ Sum	17	7	Cosine	9.980325		
½ Diff.	3	17	Sine	8.757955		
½ El. Time	11	30	Co-ar.	0.700345		
Decl. S.	19	34	Sec. less. R.	0.025833		
Constant Total, to be reserved				19.464458		
Lat. Acc. N.	48	30	Sec. less R.	0.1787		
Mid. Time	26	5	Sine	9.6431		
½ El. Time	11	30				
Remaind.	14	35		0.301030		
½ Remaind.	7	17	Sine	9.1030		
			Ditto	9.1030		
Decl. S.	19	34	Cosine	9.974167		
Lat. Acc. N.	48	30	Cosine	9.8213		
			Com. Log.	2.3025	N°	201
Great. Alt.	20	24	Sine	3.5423	N°	3486
Merid. Alt.	21	39	Sine	9.5667	Sum	3687
Decl. S.	19	34				
Co-lat.	41	13				
Latitude N.	48	47	true Latitude.			

7. By these Examples and the order of the Natural and Artificial Sines, four Figures beside Index are sufficient for giving the Sun's Meridian Altitude as near as is wanted, whilst that Altitude does not exceed Seventy Degrees; for under this it generally gives it to the nearest Minute of a Degree. If an Error of two Minutes of a Degree in the Latitude would not be thought too much, the same Number of Figures may (in usual Cases) be used for Altitudes to near Eighty Degrees; thus far Operations may be made a first, second, or any Number of times very readily with a few Figures after the constant ones are taken out at length; and then, the whole six may be applied, if the Calculation requires it.

CXX.

Having any two Altitudes of the Sun's Centre cleared from Semidiameter, Dip and Refraction, the Solar Time between the Observations, the Sun's Declination, and the Latitude by Account; to find the true Latitude.

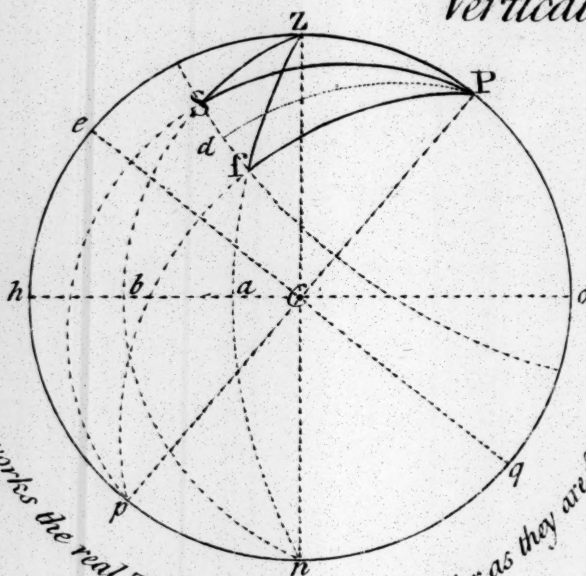
1. This Problem may be practised either at Sea or on Land, but with a Remark, that at Sea, the two Altitudes will commonly be taken, when the Ship has been removed from One Place to another, and therefore the Altitude taken at the second Place, will not be the same as it would be, if taken at the first Place of Observation. An Allowance for this Change of Place, is to be made thus, *1st.*

2. From the Place where the first Altitude is taken, suppose an horizontal Line drawn toward the

Plate 7.
Page 68

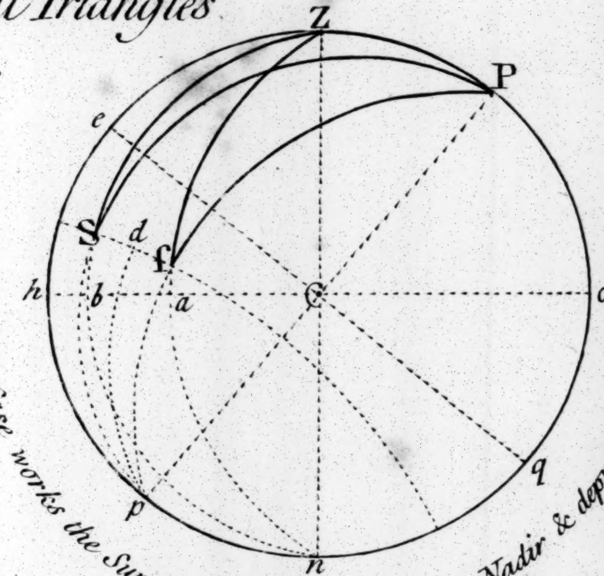
Vertical and Polar Spherical Triangles
applicable at Sea.

Case 1.



This Case works the real Triangles in the same Order as they are formed per Figure.

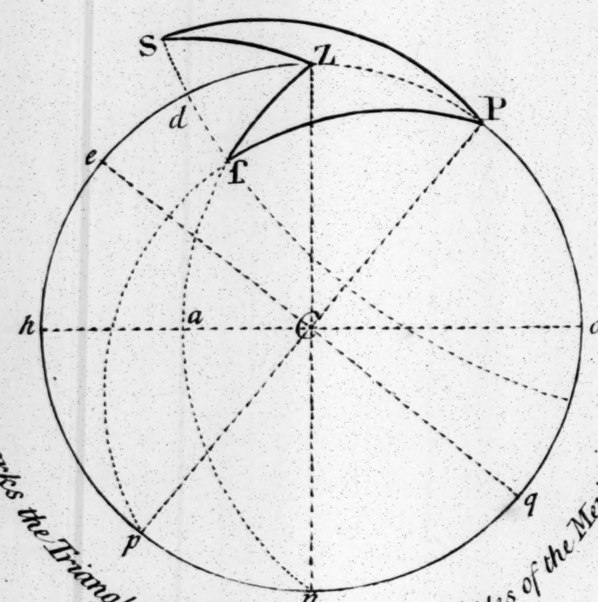
Case 2.



This Case works the Supplemental Triangles, to the Nadir & depressed Pole.

These four Cases, for taking the Times, Azimuths & Latitude; are fully explained in my Treatise of Practical Navigation.

Case 3.

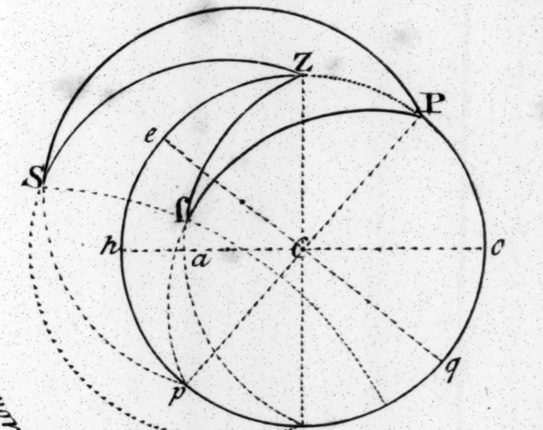


This case works the Triangles formed, but on different Sides of the Meridian of the Place.

N.B.
P denotes the elevated Pole, whether it be North or South.

Case 4.

S f is supposedly joined by the Arch of a Great Circle on which the Perp^r. from P falls.



This Case works the Supp^{tal}. Triangles on different Sides of the Meridian of the Place.

S. Dunn delin^t.

Published acording to Act of Parliament, June 14. 1786, by S. Dunn.

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Point perpendicularly under the Sun, and likewise, let it be continued behind the Ship's Place, and call it the horizontal Line of the first Observation. 2*d*. When the Ship is removed to the second Place of Observation; find, by the Course made with this Line, and the Distance sailed, the Place of the Ship at the second Place of Observation. 3*d*. The Distance sailed will be the Hypothenuse of a Right-angled Plane Triangle, the Departure from this Line will be one Leg, and the other Leg will be the number of Minutes to be either added to or subtracted from the second Altitude, to reduce it to what it would have been if taken at the first Place of Observation.

3. To exemplify these Rules, let the horizontal Bearing of the Sun, at the first Observation be supposed South South East, or any other Bearing, and on that Line will the additive or subductive Minutes be reckoned, as the Difference of Latitude is on the Meridian. The Altitude may be any thing at the first Place of Observation, but the Question is, what are the Minutes additive or subductive, for any given Distance from the first Place, and Course with this Line?

EXAMPLE I.

Forward in the Line 10 Miles.
Course with the Line 0 Degrees.
Correction subductive 10 Minutes.

EXAMPLE II.

Backward in the Line 30 Miles.
Course with the Line 0 Degrees.
Correction additive 30 Minutes.

EXAMPLE III.

Right Angles to the Line 25 Miles.
Course with the Line 90 Degrees.
Correction Nothing.

EXAMPLE IV.

Forward toward Sun 20 Miles.
Course with the Line 36 Degrees.
Correction subductive 16 Minutes.

EXAMPLE V.

Backward from Sun 40 Miles.
Course with the Line 50 Degrees.
Correction additive 26 Miles.

4. In these Cases, the Sun's Distance from the Earth being so great when compared with the Distance sailed in the Interval, the Sun's apparent removal between the two times, is as it would be in a circular Line keeping an equal Elevation above an horizontal Plane at the Place of either Observation; and therefore, the whole Difference of Altitude arises from the Curvature of the Earth or Sea.

5. Besides a correct Allowance for the Removal of the Ship in the Interval, the Observer should see that the Sextant or Quadrant to be used is adjusted as correctly as it can be; and the Watch to be used, brought to as equable a Rate of going as possible. It will be best if one of the Altitudes (at least) be taken as near the Prime Vertical or East or West Points of the Horizon as the Time and Place will admit; because, there, the Sun increases and decreases the fastest in Altitude. If either of the Altitudes be taken very near Noon, it will be a disadvantage, because near the Meridian the Increase and Decrease in Altitude is the slowest. Under other Positions, the Effect of a spheroidical Horizon, will arise from Circumstances before particularised.

6. The Elapsed Time measured between the two Observations, may be about an Hour, or less or more, at the Discretion of the Observer, taking all precaution and care that it be measured accurately; but, the shorter the Elapsed Time is, the greater attention and care should be employed to have the Altitudes accurate. In Cases where a longer Interval may be admitted, the less need there will be to be scrupulously exact in the Altitudes.

7. It is particularly to be noted, that the Sun's Declination should be known to the nearest Minute of a Degree, for the Instants when the two Altitudes are taken, and taken out from an Ephemeris, without trusting to any imperfect Tables. If the Ship has kept a Reckoning, the supposed Longitude and the Ship's Time roughly known, will give the Ephemeris's Time roughly, and from the Principles before delivered, this will be sufficiently exact for taking the Declination, in order to get the true Polar-distance.

8. If the Observer takes two Altitudes at the time of the first Observation, they will shew whether the Sun is rising or falling in Altitude, and the like may be said for the Time of the second Observation, and these will shew whether the Observations were made on one or on different Sides of the Meridian; but, there is no necessity of attending to this, because the Computation is such, that the two Hour-angles or equatorial Distances, of the Sun from Noon are here to be calculated; and in usual Cases, when the Sum of them agrees nearest with the Elapsed Time, the Observations will have been made one of them before and the other after Noon; but, when the Difference of them agrees nearest with the Elapsed Time, they will have been made both before or both after Noon.

9. The Observations being made, the Observer will have Six things given, namely, the two

Altitudes, each in Degrees and Minutes cleared from Semidiameter, Dip and Refraction; the Elapsed Time turned into Degrees and Minutes, the Sun's Declination, and consequently his Polar-distance, the Latitude by the Ship's Reckoning, either imperfectly known or supposed; and with these any other Latitude greater or less than that by the Reckoning, at Discretion; in order from them to find the true Latitude.

CXXI.

General Rules for computing the true Latitude, having the Latitude by Account, two Altitudes, the Elapsed Time, the Sun's Declination, and a newly assumed Latitude at Discretion.

1. With the Co-latitude by the Ship's Reckoning, the Polar-distance, and each of the two Co-altitudes, compute the two Hour-angles from Noon, for the Times of the two Observations, and their Sum or Difference (that of the two agreeing nearest with the Elapsed Time) will be the first Proof that the Latitude by the Reckoning, is either true or erroneous; for, if the Latitude used in these Calculations is the true Latitude, the Elapsed Time thus found will be nearly the same as that which was observed. If it be more or less, the Latitude used is not the true Latitude, and another Latitude must be used, to discover the Truth.

2. Assume another Latitude, a little greater or less than the former; and by its Co-latitude, and the other Data before mentioned, get the Elapsed Time, and it will be more or less than the first calculated Elapsed Time. Here may happen three Cases. *1st.* The measured Elapsed Time by the Watch may fall between the two computed ones. *2d.* Both computed Elapsed Times, may be greater than that measured by the Watch. *3d.* Or, both computed Elapsed Times may be less than that measured by the Watch. In either of the two latter Cases, it will appear which of the two Latitudes used is nearest the Truth; for it will be that which has its computed Elapsed Time nearest to the measured Elapsed Time.

3. Subtract one computed Elapsed Time from the other, the Remainder is to be a first Term in Direct Proportion; a second Term is the Difference of the two assumed Latitudes; and, a third Term is the Difference between the measured Elapsed Time and either of the two computed Ones; that which is for the nearest Latitude to the Truth being the most proper to be used.

4. As the Difference of the two computed Elapsed Times, is to the Difference of the two assumed Latitudes from which they were computed; so is the Difference between the measured Elapsed Time and that arising from either of the assumed Latitudes (which is here supposed to be that of the nearest to the true Latitude) to a number of Minutes of a Degree; which are to be added to or subtracted from, the nearest Latitude to Truth (when it is used, as the Case requires) and the Sum or Difference is the Latitude corrected by the first Operation.

5. With this corrected Latitude and the former Latitude that was assumed nearest the Truth, make a second Operation, and so on, till the true Latitude is thus found; or otherwise infer it by the little Differences increasing in the Approximation. Then, the Elapsed Time, with all other particulars computed therefrom, will be near the Truth.

6. Among the Copper-plate Prints in this Treatise, is one for illustrating this Method of Calculation at large, from the Original Data to the Answer. The following Example, is formed from real Observations made by me in London, when the Situation of the Sun was not under favourable Circumstances, notwithstanding which, it gives the Latitude very near the Truth.

EXAMPLE I.

Latitude by Account N.	50°	0′
Sun's Declination N.	23	27
One Altitude cleared	45	26
Other Altitude cleared	50	20
Elapsed Time measured	8	47½

Required the true Latitude?

With Latitude by Account.

Lat. Acc. N.	50°	0′		
Co-latitude	40	0	Co-ar.	0.19193
Polar-dist. N.	66	33	Co-ar.	0.03744
1st. Co-alt.	44	34		
Sum	151	7		
Half Sum	75	33	Sine	9.98604
Remainder	30	59	Sine	9.71163
			Sum	19.92704
	23	9½	Cosine	9.96352
1st. Alt. à Noon	46	19		
Lat. Acc. N.	50°	0′		
Co-latitude	40	0	Co-ar.	0.19193
Polar-dist. N.	66	33	Co-ar.	0.03744
2d. Co-alt.	39	40		
Sum	146	13		
Half Sum	73	6	Sine	9.98083
Remainder	33	26	Sine	9.74112
			Sum	19,95132
	19	0	Cosine	9.97566
2d. Alt. à Noon	38	0		
1st. Alt. à Noon	46	19		
Elapsed Time	8	19	for Lat.	50° 0′

With

With Latitude aſſumed.

Lat. aſſumed N.	52°	0′		
Co-latitude	38	0	Co-ar.	0.21066
Polar-diſt. N.	66	33	Co-ar.	0.03744
1ſt. Co-alt.	44	34		
Sum	149	7		
Half Sum	74	33	Sine	9.98401
Remainder	29	59	Sine	9.69875
			Sum	19.93086
	22	33½	Coſine	9.96543
1ſt. Alt. à Noon	45	7		
Lat. aſſumed N.	52°	0′	Co-ar.	0.21066
Co-latitude	38	0	Co-ar.	0.03744
Polar-diſt. N.	66	33		
2d. Co-alt.	39	40		
Sum	144	13		
Half Sum	72	6	Sine	9.97845
Remainder	32	26	Sine	9.72942
	18	5½	Sum	19.95597
2d. Alt. à Noon	36	11	Coſine	9.97798
1ſt. Alt. à Noon	45	7		
Elapſed Time	8	56	for Lat.	52° 0′
Elapſed Time	8	19	for Lat.	50 0
Difference	0	37	Diff.	2 0
True El. Time	8	47½		
Elapſed Time	8	56	for Lat.	52° 0′
Difference	0	8½		
As 37′ : 2° : : 8½ : :				ſub. 28′
Lat. compared with				52° 0
Lat. by 1ſt. Operation				51 32

7. In this Method, it is to be obſerved, that although the two Altitudes, the Polar-diſtance and the Elapſed Time, be each of them accurately known, the Latitude by the Ship's Reckoning may be ſuppoſed greater or leſs than the true Latitude by ſuch a number of Degrees or Minutes, that the Elapſed Time cannot be computed from it and the other Data; this at firſt ſight ſeems an Imperfection in this Method, but it is an Advantage, as will appear from the following Conſiderations.

8. If, when the Co-latitude, Polar-diſtance, and either of the Co-altitudes enter into the Queſtion, the Sum of any two of them together doth not make as many Degrees and Minutes as are in the third of them, the Queſtion is impoſſible; becauſe any two Sides of the ſpherical Triangle together, are greater than the third Side. Conſequently, the Error is in the Co-latitude and Latitude by account or aſſumed; and this inſtantly points out a greater or leſs Latitude to be taken nearer the Truth. Hence alſo, in many Caſes, an eaſy Method ariſeth of knowing how to make the Aſſumption at firſt Sight, within proper Limits at leaſt, and ſo as to come near the Truth by one Operation, with the Ship's Latitude by Account.

9. Although a ſingle Operation by this Method, has more Figures than ſome others, they are eaſily to be taken from the Tables, and ſeveral of them repeat throughout an Operation; this leſſens a great part of the labour that would otherwiſe happen. The Co-ars. and Sines, of Arches more than Ninety Degrees are taken out as inſtantly as thoſe under Ninety, from the Author's Logarithms; the Co-ar. of Co-latitude and of Polar-diſtance repeats in every Operation, and this alone takes off a fifth part of the turnings to the Logarithmic Tables. Another Circumſtance attending this Method, is the nearneſs of the Pages to each other, from which the Logarithms for both Latitudes are to be taken out; this alone if properly attended to, will (in uſual Caſes) ſhorten the Time of Operation.

10. However perfect the Latitudes of Places may be ſuppoſed in Maps and Charts that are elegantly Engraved and decorated with ornamental Compartments, there is ſufficient Reaſon to conclude that many of them want no little Correction in Latitude; this is one general Method whereby ſuch Corrections can be made, and the Latitudes of Places in the interior Parts of Countries and of Places near the Coaſts, determined ſufficiently exact for Geography and Navigation.

11. In the Lunar Method of finding the Longitude at Sea, this Method is applicable; firſt for diſcovering the Latitude, by having any two Altitudes and the Elapſed Time, and ſecondly, for diſcovering the Time at the Ship, to be compared with the Ephemeris's Time; the Sum or Difference of theſe being the Longitude.

12. It is alſo applicable for taking the Variation at Sea, at any time or place; for, the Latitude being hereby known, the true Azimuth becomes known, and the Difference between the true and Magnetic Azimuths is the Variation.

13. This Method being confined to no particular Limits of Elapſed Time; may therefore be applied univerſally, any where. Near the Equinoctial Line, the Elapſed Time between the two Obſervations, may be for any Part of the Day, under a quick Change of the Sun's Altitude, and without Error from the Earth's ſpheroidial Figure; and in high Latitudes, any Part of the Day may be employed for the Latitude by this Method; if the Latitude is wanted, to avoid Danger, paſs a Strait, or for other uſeful Purpoſes.

14. Thus may this Method be practiſed, without entering into a Conſideration of the various Caſes of Spherical Triangles, which muſt be known in going through the Solution after a direct manner;

for

for, in the preſent Method of Solution, when the Latitude is correct, the Elapſed Time computed therefrom will be nearly the ſame as that meaſured, and in other Caſes the Accuracy will depend on that of the Data, and the Reductions to the Circles of the Sphere.

CXXII.

Having one Altitude of the Sun taken ſomewhat near Noon, and another Altitude taken (on the ſame Day) at any time before or after Noon when the Sun has had no viſible Change in Declination, the Elapſed Solar Time, and the Latitude by Account; to find the true Latitude.

1. When the Latitude is to be determined at Sea by this Method, care ſhould be taken to obſerve one Altitude of the Sun, between the Time of Sun-riſing and the Time when the Sun is about Half an Hour paſt Noon at the Ship; the other Altitude may be taken at any Time between the firſt obſervation near Noon, and Sun-ſetting. The Time elapſed between the Obſervations is to be carefully meaſured; alſo, a correct Allowance in one Altitude for the Removal of the Ship. The nearer one of the Altitudes is taken to Noon, the more exact will be the Latitude.

2. Having the two Altitudes, the Elapſed Time and the Declination, the Polar-diſtance is to be found, and then the Operation with theſe and the Co-latitude by Account will be as follows.

1ſt. Add together, the Co-latitude by Account, the Polar-diſtance, and the Complement of the leſſer Altitude; take Half their Sum, and from that Half Sum ſubtract the Complement of the leſſer Altitude, to get a Remainder.

2d. Add together, the Co-ar. of the Co-latitude, the Co-ar. of the Polar-diſtance, the Sine of the Half Sum, and the Sine of the Remainder; Half the Sum of theſe four Logarithms, is the Coſine of an Arch; which Arch being doubled, is the equatorial Diſtance of the Sun from the Meridian of the Ship, at the Time of the leaſt Altitude, nearly. The Difference between this and the Elapſed Time is the equatorial Diſtance of the Sun from the Meridian of the Ship, at the time of the greateſt Altitude, nearly. In theſe Caſes, if the Latitude is nearly true, the Times will be near the Truth; but otherwiſe the true Times muſt be expected from a repeated Operation.

3d. Add together, the Sine of the greateſt Altitude's Diſtance from Noon, and the Coſine of the Declination; omit Index Ten in the Sum, the Remainder is the Sine of the firſt Arc, whoſe Secant leſs Radius take out and reſerve.

4th. Add together, the Sine of the Declination, and the reſerved Secant leſs Radius; omit Index Ten in the Sum, the Remainder is the Coſine of the ſecond Arc.

5th. Add together, the Sine of the greateſt Altitude, and the reſerved Secant leſs Radius; omit Index Ten in the Sum, the Remainder is the Coſine of the third Arc.

6th. Take the Difference between the ſecond Arc and the Co-declination, and it is the Correction. Add this Correction to the third Arch when the Polar-diſtance is leſs than Ninety Degrees; but, otherwiſe ſubtract it from the third Arc, the Sum or Difference is the Meridional Zenith-diſtance.

7th. The Sum or Difference of the Meridional Zenith-diſtance and the Declination (as in uſual Caſes) is the Latitude, by the firſt Operation. If this be near the Latitude aſſumed, it needs no farther Correction; but, when they differ much, a ſecond Operation with the corrected Latitude is to be made as before. Here follow Examples for illuſtrating this Method, by Logarithms from ſix to four Places of Figures, beſide Index.

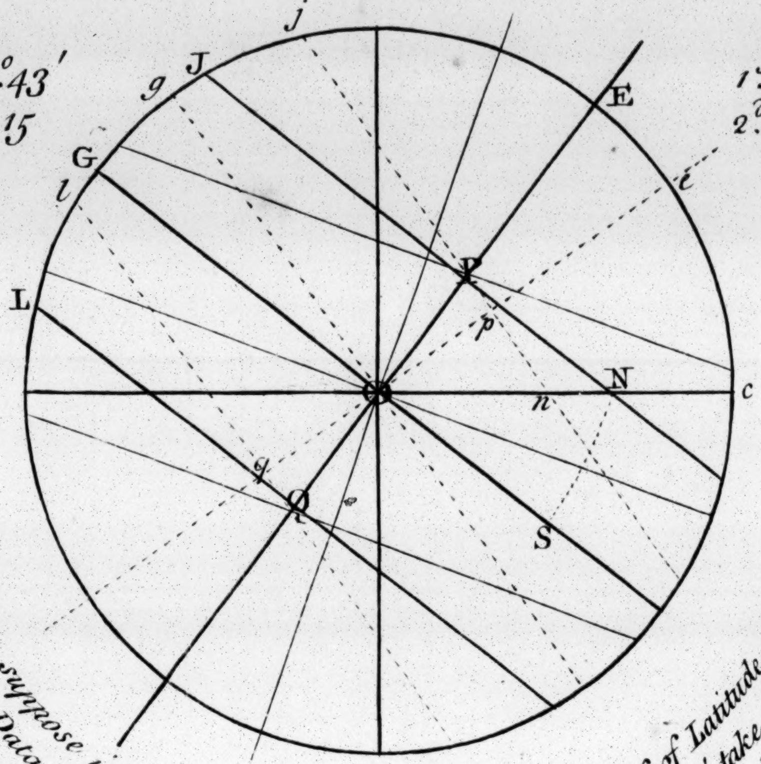
EXAMPLE I.

Lat. Acc. N.	51°	29′		
Co-lat. N.	38	31	Co-ar.	0.205692
Polar-diſt.	66	33	Co-ar.	0.037438
Co. leaſt alt.	44	34		
Sum	149	38		
Half Sum	74	49	Sine	9.984569
Remainder	30	15	Sine	9.702236
			Sum	19.929935
	22	$42\frac{1}{2}$	Coſine	9.964967
From Noon	45	25		
El. Time	8	48		
From Noon	36	37	Sine	9.775580
Decl. N.	23	27	Coſine	9.962562
Firſt Arc.	33	$10\frac{1}{2}$	Sine	9.738142
Decl. N.	23	27	Sine	9.599827
Firſt Arc.	33	$10\frac{1}{2}$	Sec. l. R.	0.077273
Second Arc.	61	$36\frac{1}{2}$	Coſine	9.677100
Great Alt.	50	20	Sine	9.886362
Firſt Arc.	33	$10\frac{1}{2}$	Secant	0.077273
Third Arc.	23	7	Coſine	19.963635

Second Arc.	61°	$36\frac{1}{2}$
Co-decl.	66	33
Correction	4	$56\frac{1}{2}$
Third Arc.	23	7
Zenith-diſt.	28	$3\frac{1}{2}$
Decl. N.	23	27
True Lat. N.	50	$30\frac{1}{2}$

EXAMPLE

Data.
Polar dist. 78°.43'
Elap. time 21.15



Data.
1st *Co alt.* 43°.5'
2d *Co alt.* 35.53

Assume the Latitude twice, as suppose it is 46°.20', or 46°.36'; with which & the Data; get the two hour Angles, & the false elapsed times will be 20°.34' & 21°.42', the diff. is 68' & the

diff. of Latitude 16'. From the true elapsed time 21°.15' take 20°.34' remains 41'. As 68 to 16', so 41' to 10', which added to 46°.20' gives 46°.30' the true Latitude of the Place required.

Calculation.

Lat.	46 . 20				Lat.	46 . 20		
Co. lat.	43 . 40	Co. ar.	0 . 16086		Co. lat.	43 . 40	Co. ar.	0 . 16086
à Pole	78 . 43	Co. ar.	0 . 00848		à Pole	78 . 43	Co. ar	0 . 00848
Co. alt	43 . 5				Co. alt.	35 . 53		
2	165 . 28				2	158 . 16		
	82 . 44	Sine	9 . 99650			79 . 8	Sine	9 . 99214
	39 . 39	Sine	9 . 80489			43 . 15	Sine	9 . 83581
		2	19 . 97073				2	19 . 99729
	14 . 47		9 . 98536			4 . 30		9 . 99864
∠	29 . 34		is 20°. 34' Diff.		∠	9 . 0		

Lat.	46 . 36				Lat.	46 . 36		
Co. lat.	43 . 24	Co. ar.	0 . 16299		Co. lat.	43 . 24	Co. ar	0 . 16299
à Pole	78 . 43	Co. ar.	0 . 00848		à Pole	78 . 43	Co. ar	0 . 00848
Co. alt	43 . 5				Co. alt	35 . 53		
2	165 . 12				2	158 . 0		
	82 . 36	Sine	9 . 99637			79 . 0	Sine	9 . 99195
	39 . 31	Sine	9 . 80366			43 . 7	Sine	9 . 83473
		2	19 . 97150				2	19 . 97815
	14 . 36		9 . 98575			3 . 45		9 . 99907
∠	29 . 12		is 21°. 42' Diff.		∠	7 . 30		

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EXAMPLE II.

First Operation.

Lat. Acc. N.	51°	32′		
Co lat. N.	38	28	Co-ar.	0.206168
Polar-dist.	72	34	Co-ar.	0.020422
Co. least Alt.	44	53		
Sum	155	55		
Half Sum	77	57	Sine	9.990324
Remainder	33	4	Sine	9.736886
			Sum	19.953800
	18	31	Cosine	9.976900
From Noon	37	2		
Eld. Time	14	41		
From Noon	22	21	Sine	9.580085
Decl. N.	17	26	Cosine	9.979578
First Arc.	21	16½	Sine	9.559663
Decl. N.	17	26	Sine	9.476536
	21	16½	Sec.l.R.	0.030654
Second Arc.	71	15	Cosine	9.507190
Gr. Alt.	51	35	Sine	9.894046
			Sec.l.R.	0.030654
Third Arc.	32	46	Cosine	9.924700
Second Arc.	71	15		
Co-decl.	72	34	Zenith-dist.	34° 5′
Correction	1	19	Decl. N.	17 26
Third Arc.	32	46	Lat. N.	51 31

This was the true Latitude.

EXAMPLE III.

First Operation.

Lat. Acc. N.	46°	20′		
Co-lat. N.	43	40	Co-ar.	0.160860
Polar-dist.	78	43	Co-ar.	0.008476
Co. least Alt.	43	5		
Sum	165	28		
Half Sum	82	44	Sine	9.996498
Remainder	39	39	Sine	9.804886
			Sum	19.970720
	14	47½	Cosine	9.985360
From Noon	29	35		
El. Time	21	51		
From Noon	8	20	Sine	9.161164
Decl. N.	11	17	Cosine	9.991524
First Arc.	8	10	Sine	9.152668
Decl. N.	11	17	Sine	9.291504
	8	10	Sec.l.R.	0.004427
Second Arc.	78	36	Cosine	9.295931
Gr. Alt.	57	7	Sine	9.908600
			Sec.l.R.	0.004427
Third Arc.	35	4	Cosine	9.913027
Second Arc.	78	36		
Co-decl.	78	43	Zenith-dist.	35° 11′
Correction	0	7	Decl. N.	11 17
Third Arc.	35	4	Lat. N.	46 28

By a second Operation, the true Lat. 46 30

EXAMPLE IV.

First Operation.

Lat. Acc. N.	51°	30′		
Co-lat. N.	38	30	Co-ar.	0.205850
Polar-dist.	66	32	Co-ar.	0.037492
Co. least alt.	52	30		
Sum	157	32		
Half Sum	78	46	Sine	9.991600
Remainder	26	16	Sine	9.645962
			Sum	19.880904
	29	19½	Cosine	9.940452
From Noon	58	39		
El. Time	60	0		
From Noon	1	21	Sine	8.372171
Decl. N.	23	28	Cosine	9.962508
First Arc	1	14	Sine	8.334679
Decl. N.	23	28	Sine	9.600118
	1	14	Sec. l. R.	0.000101
Second Arc	66	31½	Cosine	9.600219
Gr. Alt.	62	0	Sine	9.945935
			Sec. l. R.	0.000101
Third Arc	27	58	Cosine	9.946036
Second Arc	66	31½		
Co-decl.	66	32	Zenith-dist.	27° 58½′
Correction	0	0½	Decl. N.	23 28
Third Arc	27	58	Lat.	N. 51 26½

By a second Operation, the true Lat. is 51 26½

EXAMPLE V.

First Operation.

Lat. Acc. N.	5°	0′		
Co-lat. N.	85	0	Co-ar.	0.001656
Polar-dist.	88	0	Co-ar.	0.000265
Co. least alt.	74	0		
Sum	247	0		
Half Sum	123	30	Sine	9.921107
Remainder	49	30	Sine	9.881045
			Sum	19.804073
	37	3½	Cosine	9.902036
From Noon	74	7		
El. Time	75	0		
From Noon	0	53	Sine	8.187985
Decl. N.	2	0	Cosine	9.999735
First Arc	0	53	Sine	8.187720
Decl. N.	2	0	Sine	8.542819
	0	53	Sec. l. R.	0.000052
Second Arc.	88	0	Cosine	8 .542871
Gr. Alt.	86	0	Sine	9.998941
			Sec. l. R.	0.000052
Third Arc	3	54	Cosine	9 .998993
Second Arc	88	0		
Co-decl.	88	0	Zenith-dist.	3° 54′
Correction	0	0	Decl. N.	2 0
Third Arc.	3	54	True Lat. N.	5 54

Here the greatest Alt. was near the Meridian.

EXAMPLE VI.

First Operation.

Lat. Acc. N.	4°	0′		
Co-lat. N.	86	0	Co-ar.	0.001059
Polar-dist.	83	0	Co-ar.	0.003249
Co. least alt.	74	0		
Sum	243	0		
Half Sum	121	30	Sine	9.930766
Remainder	47	30	Sine	9.867631
			Sum	19.802705
	37	10½	Cosine	9.901352
From Noon	74	21		
El. Time	75	0		
From Noon	0	39	Sine	8.054781
Decl. N.	7	0	Cosine	9.996751
First Arc	0	39	Sine	8.051532
Decl. N.	7	0	Sine	9.085895
	0	39	Sec. l. R.	0.000028
Second Arc	8	30	Cosine	9.085923
Gr. Alt.	82	0	Sine	9.995753
			Sec. l. R.	0.000028
Third Arc	7	58	Cosine	9.995781
Second Arc	83	0		
Co-decl.	83	0	Zenith-dist.	7° 58′
Correction	0	0	Decl. N.	7 0
Third Arc	7	58	Lat. S.	0 58

EXAMPLE VI.

Second Operation.

Latitude S.	0°	58′		
Co-lat. S.	89	2	Co-ar.	0.000062
Polar-dist.	97	0	Co-ar.	0.003249
Co. least alt.	74	0		
Sum	260	2		
Half Sum	130	1	Sine	9.884148
Remainder	56	1	Sine	9.918659
			Sum	19.806118
	36	52½	Cosine	9.903059
From Noon	73	45		
El. Time	75	0		
From Noon	1	15	Sine	8.338753
Decl. N.	7	0	Cosine	9.996751
First Arc	1	14	Sine	8.335504
Decl. N.	7	0	Sine	9.085895
	1	14	Sec. l. R.	0.000101
Second Arc	83	0	Cosine	9.085996
Great. Alt.	82	0	Sine	9.995753
			Sec. l. R.	0.000101
Third Arc	7	54	Cosine	9.995854
Second Arc	83	0		
Co-decl.	83	0	Zenith-dist.	7° 54′
Correction	0	0	Dec. N.	7 0
Third Arc.	7	54	True Lat. S.	0 54

EXAMPLE VII.

First Operation.

Lat. Acc. N.	3°	0′		
Co-lat. N.	87	0	Co-ar.	0.000596
Polar-dist.	84	0	Co-ar.	0.002386
Co. least alt.	72	0		
Sum	243	0		
Half Sum	121	30	Sine	9.930766
Remainder	49	30	Sine	9.881045
			Sum	19.814793
	36	7	Cosine	9.907396
From Noon	72	14		
El. Time	70	0		
From Noon	2	14	Sine	8.590721
Decl. N.	6	0	Cosine	9.997614
First Arc	2	13	Sine	8.588335
Decl. N.	6	0	Sine	9.019235
	2	13	Sec. l. R.	0.000325
Second Arc	84	0	Cosine	9.019560
Great. Alt.	86	0	Sine	9.998941
			Sec. l. R.	0.000325
Third Arc	3	20	Cosine	9.999266
Second Arc	84	0		
Co-decl.	84	0	Zenith-dist.	3° 20′
Correction	0	0	Decl. N.	6 0
Third Arc	3	20	Lat. N.	2 40

EXAMPLE VII.

Second Operation.

Latitude N.	2°	40′		
Co-lat. N.	87	20	Co-ar.	0.000471
Polar-dist.	84	0	Co-ar.	0.002385
Co. least alt.	72	0		
Sum	243	20		
Half Sum	121	40	Sine	9.929989
Remainder	49	40	Sine	9.882121
			Sum	19.814967
	36	5	Cosine	9.907483
From Noon	72	10		
El. Time	70	0		
From Noon	2	10	Sine	8.577566
Decl. N.	6	0	Cosine	9.997614
First Arc	2	9	Sine	8.575180
Decl. N.	6	0	Sine	9.019235
	2	9	Sec. l. R.	0.000306
Second Arc	84	0	Cosine	9.019541
Great. Alt.	86	0	Sine	9.998941
			Secant	0.000306
Third Arc	3	23	Cosine	9.999247
Second Arc	84	0		
Co-decl.	84	0	Zenith-dist.	3° 23′
Correction	0	0	Decl. N.	6 0
Third Arc	3	23	True Lat. N.	2 37

EXAMPLE VIII.

First Operation.

Lat. Acc. N.	43°	30′		
Co-lat. N.	46	30	Co-ar.	0.139438
Polar-dist.	113	28	Co-ar.	0.037492
Co. least Alt.	80	0		
Sum	239	58		
Half Sum	113	59	Sine	9.937603
Remainder	39	59	Sine	9.807917
			Sum	19.922450
	23	51	Cosine	9.961225
From Noon	47	42		
El. Time	45	0		
From Noon	2	42	Sine	8.673080
Decl. N.	23	28	Cosine	9.962508
First Arc	2	28	Sine	8.635588
Decl. N.	23	28	Sine	9.600118
	2	28	Sec l. R.	0.000408
Second Arc	66	30½	Cosine	9 600526
Great. Alt.	22	0	Sine	9.573575
			Sec. l. R.	0.000408
Third Arc	67	58	Cosine	9.573983
Second Arc	66	30½		
Co-decl.	66	32	Zenith-dist.	67° 56½
Correction	0	1½	Decl. N	23 28
Third Arc	67	58	Lat. N.	44 28½

EXAMPLE VIII.

Second Operation.

Latitude N.	44°	28′		
Co-lat. N.	45	32	Co-ar.	0.146510
Polar dist.	113	28	Co-ar.	0.037492
Co-least Alt.	80	0		
Sum	239	0		
Half Sum	119	3	Sine	9.939697
Remainder	39	30	Sine	9.803510
			Sum	19.927209
	23	8	Cosine	9.963604
From Noon	46	16		
El. Time	45	0		
From Noon	1	16	Sine	8.344504
	23	18	Cosine	9.962508
First Arc	1	10	Sine	8.307012
Decl. N.	23	28	Sine	9.600118
	1	10	Sec. l. R.	0.000090
Second Arc	66	31½	Cosine	9.600208
Great. Alt.	22	0	Sine	9.573575
			Sec. l. R.	0.000090
Third Arc	67	59½	Cosine	9.573665
Second Arc	66	31½		
Co-decl.	66	32	Zenith-dist.	67° 59′
Correction	0	0½	Decl. N.	23 28
Third Arc	67	59½	True Lat. N.	44 31

EXAMPLE IX.

First Operation.

Lat. Acc. N.	50°	40′		
Co-lat. N.	39	20	Co-ar.	0.198027
Polar-dist.	110	0	Co-ar.	0 027014
Co. least Alt.	72	47		
Sum	222	7		
Half Sum	111	3	Sine	9.970006
Remainder	38	16	Sine	9.791917
			Sum	19.986964
	9	54	Cosine	9.993482
From Noon	19	48		
El. Time	15	0		
From Noon	4	48	Sine	8.922610
Decl. S.	20	0	Cosine	9.972986
First Arc.	4	31	Sine	8.895596
Decl. S.	20	0	Sine	9.534052
	4	31	Sec.l.R.	0.001351
Second Arc.	69	56	Cosine	9.535403
Great. Alt.	19	41	Sine	9.527400
			Sec.l.R	0.001432
Third Arc.	70	15	Cosine	9.528832
Second Arc.	69	56		
Co-decl.	70	0	Zenith-dist.	70° 11′
Correction	0	4	Decl. S.	20 0
Third Arc.	70	15	Lat. N.	50 11

EXAMPLE IX.

Second Operation.

Latitude N.	50°	11′		
Co-lat. N.	39	49	Co-ar.	0.193594
Polar-dist.	110	0	Co-ar.	0.027014
Co-least. Alt.	72	47		
Sum	222	36		
Half Sum	111	18	Sine	9.969272
Remainder	38	31	Sine	9.794308
			Sum	19.984188
	10	54	Cosine	9.992094
From Noon	21	48		
El. Time	15	0		
From Noon	6	48	Sine	9.073366
Decl. S.	20	0	Cosine	9.972986
First Arc	6	23	Sine	9.046352
Decl. S.	20	0	Sine	9.534052
	6	23	Sec.l.R.	0.002701
Second Arc	69	52½	Cosine	9 536753
Great. Alt.	19	41	Sine	9 527400
			Sec.l.R	0.002701
Third Arc	70	11	Cosine	9.530101
Second Arc	69	52½		
Co-decl.	70	0	Zenith-dist.	70° 3′½
Correction	0	7½	Decl. S.	20 0
Third Arc	70	11	Lat. N.	50 3½
By the First Operation			Lat N.	50 11

By approaching Differences, true Lat. 50 0

EXAMPLE X.

Firſt Operation.

Lat. Acc. N.	49° 10′		
Co-lat. N.	40 50	Co-ar.	0.184515
Polar-diſt.	110 0	Co-ar.	0.027014
Co-leaſt Alt.	72 47		
Sum	223 37		
Half Sum	111 48	Sine	9.967775
Remainder	39 1	Sine	9.799028
		Sum	19.978332
	12 45	Coſine	9.989166
From Noon	25 30		
El. Time	15 0		
From Noon	10 30	Sine	9.260633
Decl. S.	20 0	Coſine	9.972986
Firſt Arc	9 52	Sine	9.233619
Decl. S.	20 0	Sine	9.534052
	9 52	Sec. l. R.	0.006472
Second Arc	69 41	Coſine	9.540524
Great. Alt.	19 41	Sine	9.527400
		Sec. l. R.	0.006472
Third Arc	70 1	Coſine	9.533872
Second Arc	69 41		
Co-decl.	70 0	Zenith-diſt.	69° 42′
Correction	0 19	Decl. S.	20 0
Third Arc	70 1	Lat. N.	49 42

EXAMPLE X.

Second Operation.

Latitude N.	49° 42′		
Co-lat. N.	40 18	Co-ar.	0.189237
Polar-diſt.	110 0	Co-ar.	0.027014
Co-leaſt Alt.	72 47		
Sum	223 5		
Half Sum	111 32	Sine	9.968578
Remainder	38 45	Sine	9.796521
		Sum	19.981350
	11 50	Coſine	9.990675
From Noon	23 40		
El. Time	15 0		
From Noon	8 40	Sine	9.178072
Decl. S.	20 0	Coſine	9.972986
Firſt Arc	8 8½	Sine	9.151058
Decl. S.	20 0	Sine	9.534052
	8 8½	Secant	0.004400
Second Arc	69 47	Coſine	9.538452
Great. Alt.	19 41	Sine	9.527400
		Sec. l. R.	0.004400
Third Arc	70 6	Coſine	9.531800
Second Arc	69 47		
Co-decl.	70 0	Zenith diſt.	69° 53′
Correction	0 13	Decl. S	20 0
Third Arc	70 6	Lat. N.	49 53
By the firſt Operation		Lat. N.	49 42

By approaching Differences, true Lat. 50 2

EXAMPLE XI.

Firſt Operation.

Lat. Acc N.	47° 24′		
Co-lat. N.	42 36	Co-ar.	0.169491
Polar-diſt.	77 40	Co-ar.	0.010140
Co-leaſt Alt.	40 42		
Sum	160 58		
Half Sum	80 29	Sine	9.993981
Remainder	39 47	Sine	9.806103
		Sum	19.979715
	12 20½	Coſine	9.989857
From Noon	24 41		
El. Time	42 30		
From Noon	17 49	Sine	9.485682
Decl. N.	12 20	Coſine	9.989860
Firſt Arc	17 23½	Sine	9.475542
Decl. N.	12 20	Sine	9.329599
	17 23½	Sec. l. R.	0.020322
Second Arc	77 3½	Coſine	9.349921
Great. Alt.	52 4	Sine	9.896926
		Sec. l. R.	0.020322
Third Arc	34 15½	Coſine	9.917248
Second Arc	77 3½		
Co-decl.	77 40	Zenith-diſt.	34° 52′
Correction	0 36½	Decl. N.	12 20
Third Arc	34 15½	Lat N.	47 12

EXAMPLE XI.

Second Operation.

Latitude N.	47° 12′		
Co-lat. N.	42 48	Co-ar.	0.167842
Polar-diſt.	77 40	Co-ar.	0.010140
Co-leaſt Alt.	40 42		
Sum	161 10		
Half Sum	80 35	Sine	9.994108
Remainder	39 53	Sine	9.807011
		Sum	19.979107
	12 31½	Coſine	9.989553
From Noon	25 3		
El. Time	42 30		
From Noon	17 27	Sine	9.476938
Decl. N.	12 20	Coſine	9.989860
Firſt Arc	17 2	Sine	9.466798
Decl. N.	12 20	Sine	9.329599
	17 2	Sec. l. R.	0.019481
Second Arc	77 5½	Coſine	9.349080
Great. Alt.	52 4	Sine	9.896926
		Sec. l. R.	0.019481
Third Arc	34 25	Coſine	9.916407
Second Arc	77 5½		
Co-decl.	77 40	Zenith-diſt.	34° 59½′
Correction	0 34½	Decl. N.	12 20
Third Arc	34 25	Lat. N.	47 19½
By the firſt Operation		Lat. N.	47 12

By the Medium of the two, true Lat. 47 16

EXAMPLE XII.

First Operation.

Lat. Acc. N	49°	17′		
Co-lat. N.	40	43	Co-ar.	0.185540
Polar-dist.	113	28	Co-ar.	0.037492
Co. least Alt.	77	11		
Sum	231	22		
Half Sum	115	41	Sine	9.954823
Remainder	38	30	Sine	9.794150
			Sum	19.972005
	14	27	Cosine	9.986002
From Noon	28	54		
El. Time	55	0		
From Noon	26	6	Sine	9.643393
Decl. S.	23	28	Cosine	9.962508
First Arc	23	48	Sine	9.605901
Decl. S.	23	28	Sine	9.600118
	23	48	Sec. l. R.	0.038598
Second Arc	64	12	Cosine	9.638716
Great. Alt.	14	15	Sine	9.391206
			Sec. l. R.	0.038598
Third Arc	74	23½	Cosine	9.429804
Second Arc	64	12		
Co-decl.	66	32	Zenith-dist.	72° 3½′
Correction	2	20	Decl. S.	23 28
Third Arc	74	23½	Lat. N.	48 35½

EXAMPLE XII.

Second Operation.

Lat. Acc. N.	48°	35′		
Co-lat. N.	41	25	Co-ar.	0.179450
Polar-dist	113	28	Co-ar.	0.037492
Co-least Alt.	77	11		
Sum	232	4		
Half Sum	116	2	Sine	9.953537
Remainder	38	51	Sine	9.797404
			Sum	19.967943
	15	28½	Cosine	9.983971
From Noon	30	57		
El. Time	55	0		
From Noon	24	3	Sine	9.610163
Decl. S.	23	28	Cosine	9.962508
First Arc	21	57	Sine	9.572671
Decl. S.	23	28	Sine	9.600118
	21	57	Sec. l. R.	0.032681
Second Arc	64	34½	Cosine	9.632799
Great. Alt.	14	15	Sine	9.391206
			Sec. l. R.	0.032681
Third Arc	74	36½	Cosine	9.423887
Second Arc	64	34½		
Co-decl.	66	32	Zenith-dist.	72° 39′
Correction	1	57½	Decl. S.	23 28
Third Arc	74	36½	Lat. N.	49 11

Medium of the two Operations Lat. 48 53¼
This is but a Minute less than the Truth.

3. From these Examples, it is evident that this Method approximates toward the Truth in each Operation; but, that the relation of the parts of the Data to each other, may be such, as either to give the Result greater or less, or alternatively greater and less than the Truth, at the End of each Operation. Thus, in Example the third, the Approximation is by a continued Increase; in the fourth it is a Confirmation; the Sun was very near the Meridian, at the times of greatest Altitudes, in the fifth and sixth Examples; in the seventh it is by a continued decrease; in the eighth by continued Increase; in the ninth the like; in the tenth by continued increase; in the eleventh by increase and decrease; and the like in the twelfth Example.

4. It is also evident that, the approach toward the Truth, is in some instances greater by a single Operation, than in others; and therefore, the Terms in the Data may sometimes be so related to each other in the extreme Limits of the approximation, as to admit of no Convergency, and beyond these Limits a Divergency may arise so as to render the Operation insufficient, although within proper Limits it may readily converge to near the Truth. In such Cases, due regard should be had to the Design of the Operation, and and the Data that are proper for bringing out a true Result.

5. Thus, in the foregoing Examples, the small equatorial Distance of the Sun from the Meridian at the time of greatest Altitude, is what makes the Operation converge quickly toward the Truth; and the reverse, from a great Distance. The Examples, Distances in whole Degrees, and Tendency; are nearly as follows.

Examp.	Page.	Dist.	Convergency.
1	72	36½ Degrees,	slow.
2	73	22½ Degrees,	slow.
3	73	8 Degrees,	quick.
4	73	1½ Degree,	very quick.
5	73	1 Degree,	very quick.
6	74	1¼ Degree,	very quick.
7	74	2¼ Degrees,	quick.
8	75	1¼ Degree,	very quick.
9	75	6¾ Degrees,	quick.
10	76	8¼ Degrees,	quick.
11	76	17½ Degrees,	slow.
12	77	24 Degrees,	slow.

6. Having the Observations properly made, in order to make the Operation for the Correction, it is necessary to take notice of the Number of Figures that are absolutely wanted for finding the three Arcs in this Method, to the nearest Half-minute of a degree; because, on them the Answer depends.

depends. In the foregoing Examples, the Logarithms are taken out to six Figures beside Index, not as Examples for all others to be made to as many Places of Figures, it being evident that five, and frequently four Figures beside Index may be sufficient for general Use; but, this will be easily known by the Logarithms for the Arcs, as they arise. If four Figures be used to get the first Arc, and its Cosine is so great as to require five or even a sixth Figure, they may be readily added; and the like may be said for the second and third Arcs. This will in many Cases, make the Calculation more concise and equally certain.

7. When it is known from two Operations, that the Question is under the increasing, decreasing, or alternative Circumstances, it will be but seldom necessary to make a third Operation; for, under the two former, the Ratio between the two Corrections will lead nearly to the Truth; and under the latter the Truth will be nearly a Medium of the two Corrections. If one Altitude is taken near the Meridian (and many such can be had) Tediousness in the Process by this Method, will vanish, and the first Operation will go directly to near the Truth, if not Truth itself.

8. Consequently, all the principal Fixed Stars and Primary Planets may be observed, and the Latitude deduced from them by this Method; as their Situations are various in the Heavens, and adapted for all times and places on Land and at Sea.

CXXIII.

Having one Altitude of the Sun taken somewhat near Noon and another Altitude taken (on the same Day) at any time before or after Noon when the Sun has had a visible Change in Declination, the Elapsed Solar Time, and the Latitude by Account; to find the true Latitude.

1. In this Problem, the Data are the same as the former with all its Cases, excepting that in this, the Declination is given for each of the two times of Observation; and then, the Operation will be thus.

2. Having the Declination at the time of the least Altitude, get its Polar-distance, with which and the other Data get the equatorial Distance at that time from Noon; and as before, the equatorial Distance at the time of greatest Altitude, from Noon. Next, proceed with the Declination at the time of greatest Altitude from Noon, and thereby get the three Arcs, which proceed with as before, until the Latitude is had. The like for a second Operation, and Comparisons of the Corrections, until the true Latitude is discovered.

3. The quickest Change of the Sun's Declination, is near the time of the Equinoxes, and if the Latitude be sought near that time by this Method, the Interval may be six Hours, and the Sun may alter in Declination near six Minutes in that Interval. Near the time of the Solstices, the Change is small. At other times, the Alteration will depend on the Sun's distance from the solsticial Point, and the Interval or Elapsed Time.

4. Amongst the former Examples, take the fifth, when the Elapsed Time was five Hours, with a Change of Declination near five Minutes in that Interval. Here, let the second Operation be repeated with a Polar-distance five Minutes different from that of the first Operation, and it will be thus.

EXAMPLE V. Page 73.

Second Operation.

Latitude N.	5°	54′		
Co-lat. N.	84	6	Co-ar.	0.002307
Polar-dist.	88	5	Co-ar.	0.000243
Co. least Alt.	74	0		
Sum	246	11		
Half Sum	123	5½	Sine	9.923140
Remainder	49	5½	Sine	9.878383
			Sum	19.804073
	37	3¼	Cosine	9.902036
From Noon	74	6½	nearly as before.	

Here, the Change of Declination has but a small Effect on the deduced Latitude, and would have less Effect if a Medium of the two Latitudes were used instead of the Extremes. But,

5. If this Method is applied for finding the Latitude by two Altitudes of the Moon and Elapsed Time, the Moon's Altitudes must be cleared from Semidiameter, Dip, Refraction and Parallax; also, the Elapsed Time must be reduced to the Moon's Change of Right Ascension in the Interval. The Change of Declination may happen to be five Degrees or more in a Day, which is sixty-two Minutes in five Hours; then it will be thus.

Latitude N.	5°	54′		
Co-lat. N.	84	6	Co-ar.	0.002307
Polar-dist.	86	58	Co-ar.	0.000609
Co. least Alt.	74	0		
Sum	245	4		
Half Sum	122	32	Sine	9.925868
Remainder	48	32	Sine	9.874680
			Sum	19.803464
	37	6½	Cosine	9.901732
From Noon	74	13		
El. Time	75	0		
From Noon	0	47	Sine	8.135810

Decl.

Decl. N.	3	2	Cosine	9.999391
First Arc	0	47	Sine	8.135201
Decl. N.	3	2	Sine	8.723595
	0	47	Sec. I. R.	0.000041
Second Arc	86	58	Cosine	8.723636
Great. Alt.	86	0	Sine	9.998941
			Sec. I. R.	0.000041
Third Arc	3	55	Cosine	9.998982
Second Arc	86	58		
Co-decl.	86	58	Zenith-dist.	3° 55′
Correction	0	0	Decl. N.	3 2
Third Arc	3	55	Lat. N.	6 57

This is more than a Degree of Latitude, more than would arise without allowance for Change in Declination.

6. If Example the eleventh be operated by this Method with allowance of three Minutes for Change of Declination, it will be thus.

EXAMPLE XI. Page 76.

Second Operation.

Latitude N.	47°	12′		
Co-lat. N.	42	48	Co-ar.	0.167848
Polar-dist.	77	43	Co-ar.	0.010058
Co. least Alt.	40	42		
Sum	161	13		
Half Sum	80	36½	Sine	9.994140
Remainder	39	54½	Sine	9.807238
			Sum	19.979284
	12	27½	Cosine	9.989642
From Noon	24	55		
El. Time	42	30		
From Noon	17	35	Sine	9.480140
Decl. N.	12	20	Cosine	9.989860
First Arc	17	10	Sine	9.470000
Decl. N.	12	20	Sine	9.323780
	17	10	Sec. I. R.	0.019792
Second Arc	77	15½	Cosine	9.343572
Great. Alt.	52	4	Sine	9.896926
			Sec. I. R.	0.019792
Third Arc	34	21½	Cosine	9.916718
Second Arc	77	15½		
Co-decl.	77	40	Zenith-dist.	34° 46′
Correction	0	24½	Decl. N.	12 20
Third Arc	34	21½	Lat. N.	47 6

This gives more than three times the Change in Latitude, that there is in the Polar-distance; the former being three, and the latter ten Minutes of a Degree.

7. The proportional and irregular Changes, which arise after a Correction is made in the Declination or Polar-distance, depend on the Relation of the Sides (and consequently of the Angles) of the original spherical Triangle, as it is formed by the Circles of the Celestial Sphere; if these approach nearly to an Equality, the little Differences formed by their Variation will be nearly similar to them; but, otherwise a small change or alteration in one may produce a much greater or less alteration in the other, whether it belong to Side or Angle, according to the Positions of the great Circles which form the Sides and Angles, and the Quantities to be varied.

8. As the Moon's Altitude may be taken at Sea in the Day-time, when not very far from the Meridian either past or short of it, this Method is applicable at such times for finding the Latitude; but, the Parallax and other Obstructions must be first cleared, according to the Directions for that Purpose. Were these the only Uses of this Problem, they might be ranked amongst those of no little Consequence in Nautical Astronomy; but, this Problem is of the most ample Extent, as will appear by the next following.

CXXIV.

In an unknown Latitude, having the Altitudes of two known Fixed Stars, one of them being not far from the Meridian of the Place of Observation; to determine the true Latitude.

1. When the Latitude is to be found by this Method, care should be taken to observe when one of the Stars is not very far from the Meridian of the place of Observation, either past or short of it; the other Star may be any other how situated.

2. If the Place of Observation is in North Latitude, the northern Pole-star, will appear above the Horizon (if it is not effaced by the horizontal Vapours) some time or other in the Night, and that may be an imperfect Guide for knowing the north Point of the Horizon within one or two Degrees; but, if that Star cannot be seen, the north Part of the Horizon may be found nearly, thus.

3. Find the four Stars α, β, γ, δ, in the Square of the Constellation Ursa Major (the Great Bear). Of these, α and β are commonly called the two Pointers, because a line produced from β through α points toward the Pole-star, and the Pole itself is not quite two Degrees from the Pole. The Stars α and δ are nearest to the Pole-star; β and γ are farthest from it.

4. Imaginarily, divide the Distance from α to δ into Eight equal Parts, and the Distance from β to γ into Four equal Parts, and on these two Lines, take from α to δ one Eighth of its Distance, and from β to γ one Fourth of its Distance; these two Points will be nearly in the Meridian

 TWO ALTITUDES.

Meridian of the Pole-ſtar, but that Star will be on the other Side of the Meridian, beyond the true Pole.

5. From the Pole-ſtar, direct your Sight toward the neareſt of theſe Points, near two degrees, and that is the Place of the true Pole.

6. At Sea, let two Perſons either ſtand, or hold a Cord, Rod, or any other ſtrait thing in that Direction; ſo the northermoſt Perſon will ſee what Stars are paſt the Meridian of the Place of Obſervation and what are ſhort of it, without any other help, when thoſe Stars are viſible, and well ſituated either above or below the Pole.

7. By the ſame Method, it may be known what Stars are nearly Eaſt or Weſt; alſo, by this Method it may be known what the Variation of the Compaſs is without erring many Degrees at all Hours when thoſe Stars appear; and, if ſuch obſervations are made when the Pole itſelf is on the Meridian, the Variation may be taken near the truth, by Inſpection.

8. By this Method or any other, being able to judge of the Meridian's Poſition, it will be eaſy to perceive when the Star neareſt it is not many Degrees therefrom, either paſt or ſhort of it (no matter which of the two) then the Operation for the Latitude will be thus. *1ſt.*

9. Aſſume any Latitude at pleaſure as though it was a Latitude by Account. *2d.* Add together the Co-latitude aſſumed, the Polar-diſtance of the Star fartheſt from the Meridian, the Co-altitude of the Star fartheſt from the Meridian, and from their Half Sum ſubtract the Co-altitude of the Star fartheſt from the Meridian, to get a Remainder. *3d.* Add together the Co-ar. of the Co-latitude aſſumed, the Co-ar. of the Polar-diſtance of the Star fartheſt from the Meridian, the Sine of the Half Sum and the Sine of the Remainder, Half the Sum of theſe four Logarithms is the Coſine of an Arch, which doubled, is the approximated Diſtance of the fartheſt Star from the Meridian. *4th.* By the Right Aſcenſions of the two Stars, get their Equatorial Diſtance in Degrees and Minutes. This is done by ſubtracting the Right Aſcenſion of one from that of the other; but, if the beginning of the Ecliptic is between their Meridians, then take the greateſt Right Aſcenſion from three hundred and ſixty Degrees, and add the Remainder to the leaſt Right Aſcenſion (this is beſt ſhewn by Diagram). *5th.* Take the Difference between the approximated Diſtance of the fartheſt Star from the Meridian, and the Star's Equatorial Diſtance, the Remainder is the approximated Diſtance of the neareſt Star, from the Meridian. *6th.* Add together, the Sine of the neareſt Star's approximated Diſtance from the Meridian, and the Coſine of its Declination, the Sum (rejecting Ten) is the Sine of the firſt Arc. *7th.* Add together, the Sine of the Star's Declination neareſt the Meridian, and the Secant leſs Radius of the firſt Arc, the Sum (rejecting Ten) is the Coſine of the ſecond Arc. *8th.* Add together, the Sine of the Star's Altitude neareſt the Meridian, and the Secant leſs Radius of the firſt Arc, the Sum (rejecting Ten) is the Coſine of the third Arc. *9th.* The Difference between the ſecond Arc and the Co-declination of the Star neareſt the Meridian, is the Correction, to be added to the third Arc when the Polar-diſtance of the Star fartheſt from the Meridian is leſs than Ninety Degrees, but otherwiſe to be ſubtracted; this gives the Meridional Zenith-diſtance of the Star neareſt the Meridian; to which apply its Declination (as uſual) and it gives the Latitude, by the firſt Operation.

With this Latitude and the other Data, make a ſecond Operation, and a third if neceſſary; but, in uſual Caſes one Operation approaches much toward the true Latitude.

10. That this Problem is limited and will give but one particular Anſwer to the Data, is evident from the following Conſiderations. At the time when the Altitudes of the two Stars are taken, the Polar-diſtances of the two Stars, their Co-altitudes, and the true Co-latitude, do form the Sides of two ſpherical Triangles, whoſe Angles with the Meridian are at the elevated Pole. Conſequently, the two Polar-diſtances, the two Co-altitudes, and the true Co-latitude, with the whole Equatorial Diſtance; do determine the two Polar-angles, and the true Co-latitude; or in other Words, the Times of Obſervation and the Latitude of the Place.

The following is an Example, where a Ship is ſuppoſed to have loſt her Latitude and found it accurately by this Method.

EXAMPLE I.

At Sea, Latitude entirely unknown.

Lat. N. ſuppoſed about		20°	0′
Sirius near Meridian,	Alt.	32	33
Rigel fartheſt,	Alt.	35	31
Sirius, Right Aſcenſion		98	57
Rigel, Right Aſcenſion		76	4
Equatorial Diſtance		22	53
Sirius, Declination S.		16	25
Rigel, Declination S.		8	27
Sirius, Polar-diſtance		106	25
Rigel, Polar-diſtance		98	27

First Operation.

Lat. Acc. N	20°	0′		
Co-lat. N.	70	0	Co-ar.	0.0271
Rigel Pol. dift.	98	57	Co-ar.	0.0053
Rigel Co. alt.	54	29		
Sum	223	26	Sine	9.9680
Half Sum	111	43	Sine	9.9247
			Sum	19.9251
	23	28	Cofine	9.9625
Rigel à Merid.	46	56		
Equator-dift.	22	53		
Sirius à Merid.	24	3	Sine	9.6101
Sirius Decl.	16	25	Cofine	9.9819
Firft Arc	23	0	Sine	9.5920
Sirius Decl.	16	25	Sine	9.4512
	23	0	Sec. l. R.	0.0359
Second Arc	72	7	Cofine	9.4871
Sirius Alt.	32	33	Sine	9.7308
			Sec. l. R.	0.0359
Third Arc	54	14	Cofine	9.7667
Second Arc	72	7		
Sirius Co-decl.	73	35	Zenith-dift.	52°46′
Correction	1	28	Sirius Decl.	16 25
Third Arc	54	14	Lat. N.	36 21
By firft Suppofition,			Lat. N.	20 0
			Correction	16 21

A Correction of Nine hundred eighty one Miles.

Second Operation.

Lat. Acc. N.	36°	20′		
Co-lat. N.	53	40	Co-ar.	0.0939
Rigel Pol. dift.	98	57	Co-ar.	0.0053
Rigel Co. alt.	54	29		
Sum	207	6		
Half Sum	103	33	Sine	9.9877
Remainder	49	4	Sine	9.8782
			Sum	19.9651
	16	8	Cofine	9.9825
Rigel à Merid.	32	16		
Equator-dift.	22	53		
Sirius à Merid.	9	23	Sine	9.2123
Sirius Decl.	16	25	Cofine	9.9819
Firft Arc	9	0	Sine	9.1942
Sirius Decl.	16	25	Sine	9.4512
	9	0	Sec. l. R.	0.0054
Second Arc	73	22	Cofine	9.4566
Sirius Alt.	32	33	Sine	9.7308
			Sec. l. R.	0.0054
Third Arc	57	0	Cofine	9.7362
Second Arc	73	22		
Sirius Co-decl.	73	35	Zenith-dift.	56° 47′
Correction	0	13	Sirius Decl.	16 25
Third Arc	57	0	Lat. N.	40 22
By fecond Suppofition,			Lat. N.	36 20
			Correction	4 2

A Correction of Two hundred forty two Miles.

Third Operation.

Lat. Acc. N.	40°	0′		
Co-lat. N.	50	0	Co-ar.	0.1157
Rigel Pol. dift.	98	57	Co-ar.	0.0053
Rigel Co-alt.	54	29		
Sum	203	26	Sine	9.9908
Half Sum	101	43	Sine	9.8658
			Sum	19.9776
	12	56	Cofine	9.98884
Rigel à Merid.	25	52		
Equator-dift.	22	53		
Sirius à Merid.	2	59	Sine	8.7164
Sirius Decl.	16	25	Cofine	9.9819
Firft Arc	2	53	Sine	8.6983
Sirius Decl.	16	25	Sine	9.4512
	2	53	Sec. l. R.	0.0005
Second Arc	73	33½	Cofine	9.4517
Sirius Alt.	32	33	Sine	9.7308
			Sec. l. R.	0.0005
Third Arc	57	24½	Cofine	9.73136
Second Arc	73	33½		
Sirius Co-decl.	73	35	Zenith-dift.	57° 23′
Correction	0	1½	Sirius Decl.	16 25
Third Arc	57	24½	Lat. N.	40 58
			Correction	0 58

This is but four Minutes lefs than the Truth, thus.

Fourth Operation.

Lat. Acc. N.	40°	58′		
Co-lat. N.	49	2	Co-ar.	0.122001
Rigel Pol. dift.	98	57	Co-ar.	0.005320
Rigel Co-alt.	54	29		
Sum	202	28		
Half Sum	101	14	Sine	9.991600
Remainder	46	45	Sine	9.862471
			Sum	19.981392
	11	49	Cofine	9.990696
Rigel à Merid.	23	30		
Equator-dift.	22	53		
Sirius à Merid.	0	45	Sine	8.116926
Sirius Decl.	16	25	Cofine	9.981924
Firft Arc	0	43	Sine	8.098850
Sirius Decl.	16	25	Sine	9.451204
	0	43	Sec. l. R.	0.000034
Second Arc.	73	35	Cofine	9.451238
Sirius Alt.	32	33	Sine	9.730811
			Sec. l. R.	0.000034
Third Arc	57	27	Cofine	9.730845
Second Arc	73	35		
Sirius Co-decl.	73	35	Zenith-dift.	57° 27′
Correction	0	0	Sirius Decl.	16 25
Third Arc.	57	57	True Lat. N.	41 2

Whole Correction in Latitude,
One thoufand two hundred fixty two Miles.

EXAMPLE II.

At Sea, the Latitude unknown.
Aldebaran near the Meridian.
Altitude of Aldebaran 56° 2′
Altitude of Pollux 46 40
Latitude assumed N. 40 0

Co-lat. N.	50° 0′	Co-ar.	0.1157
Pollux Pol. dist.	61 28	Co-ar.	0.0562
Pollux Co-alt.	43 20		
Sum	154 48		
Half Sum	77 24	Sine	9.9894
Remainder	34 4	Sine	9.7483
		Sum	19.9096
	25 41	Cosine	9.9548
Pollux à Merid.	51 22		
Equator. dist.	48 10		
Aldeb. à Merid.	3 12	Sine	8.7468
Aldeb. Decl.	16 4	Cosine	9.9827
First Arc	3 5	Sine	8.7295
Aldeb. Decl.	16 14	Sine	9.4421
	3 5	Sec. l. R.	0.0006
Second Arc	73 55	Cosine	9.4427
Aldeb. Alt.	56 2	Sine	9.9187
		Sec. l. R.	0.0006
Third Arc	33 51	Cosine	9.9193
Second Arc	73 55		
Aldeb. Co-dec.	73 56	Zenith-dist.	33° 52′
Correction	0 1	Aldeb. Decl.	16 4
Third Arc.	33 51	Lat. N.	49 56

Second Operation.

Co-Lat. N.	40° 4′	Co-ar.	0.191331
Pollux Pol. dist.	61 28	Co-ar.	0.056239
Pollux Co-alt.	43 20		
Sum	144 52		
Half Sum	72 26	Sine	9.979260
Remainder	29 6	Sine	9.686936
		Sum	19.913766
	25 6½	Cosine	9.956883
Pollux à Merid.	50 13		
Equator-dist.	48 10		
Aldeb à Merid.	1 55	Sine	8.524343
Aldeb. Decl.	16 4	Cosine	9.982696
First Arc	1 50½	Sine	8.507039
Aldeb. Decl.	16 4	Sine	9.442096
	1 50½	Sec. l. R.	0.000224
Second Arc	73 55½	Cosine	9.442320
Aldeb. Alt.	56 2	Sine	9.918745
		Sec. l. R.	0.000224
Third Arc.	33 55½	Cosine	9.918969
Second Arc.	73 55½		
Aldeb. Co-decl.	73 56	Zenith-dist.	33° 56′
Correction	0 0½	Aldeb. Decl.	16 4
Third Arc.	33 55½	True Lat. N.	50 0

Lat. corrected, Two thousand four hundred Miles.

11. If the Precepts and Operations in this Section be compared with those of the two foregoing ones, it will be evident that the Corrections in each depend on the same Principles. In the former, two different Altitudes, the Elapsed Time, two Polar-distances, and the Latitude by Account, were the Data. In this, the Altitudes of two Stars, their equatorial-distance, their two Polar-distances, and the Latitude by Account are the Data. Therefore, the like Advantages that arise in the Convergency toward the true Latitude in one of these Methods, do arise in the other of them. The assumed Latitudes, true Latitudes, Corrections, and Equatorial Distances of the Sun from the Meridian at the Time of greatest Altitude, were thus, in the twelve Examples of Section 122.

	Assumed	True Lat.	From Merid.
1	51° 29′ N.	*51° 30′½ N.	36° 37′
2	51 32 N.	51 31 N.	22 21
3	46 20 N.	46 30 N.	8 20
4	51 30 N.	51 26½ N.	1 21
5	5 0 N.	5 54 N.	0 53
6	4 0 N.	0 54 S.	1 15
7	3 0 N.	2 37 N.	0 23
8	43 30 N.	44 31 N.	1 16
9	50 40 N.	50 11 N.	6 48
10	49 10 N.	50 2 N.	10 3
11	47 24 N.	47 16 N.	17 27
12	49 17 N.	48 53 N.	25 5

12. By comparing these Numbers, it is evident that the Convergency depends on the nearness of one of the Stars to the Meridian of the Place of Observation, and that when such Distance doth not exceed Half an Hour of Time, the Operation converges with all desireable quickness toward the Truth. In the two foregoing Examples, Sirius and Aldebaran were each but two Degrees from the Meridian, this produced a very swift Convergency. By the third Example, Half an Hour's Distance from the Meridian, will not converge slowly, this gives an Hour for judging when the Luminary is nearest the Meridian; and, there being proper Helps for judging within that Distance, the Convergency will be swifter, if the Altitude of the Star nearest the Meridian be taken thereby. On the contrary, if the Star nearest the Meridian be an Hour therefrom, which gives two Hours for the Interval, the Operation will converge, though not so swiftly, and the numerous Opportunities of this kind that happen, and much more advantageously, make ample Amends for applying this Method to remoter Distances.

* Read True Lat. N. 51 30½ instead of 50 30½, page 72.

CXXV.

Having the Altitude of the Sun and the Altitude of the Moon, taken at the same time when either of them is not far from the Meridian of the Place of Observation, the observed Distance of their Limbs, and an Ephemeris predicting the Moon's Place in the Heavens; to find the Latitude and Longitude of the Place of Observation.

1. In order to solve this Problem, it is necessary first to compute the Time at the Place the Ephemeris was made for, by having the three cotemporary Observations. This has been fully illustrated in Sections 104, and 108 and 109.

2. Having the Ephemeris Time, it gives the Sun's true Right Ascension and Declination for that Time; and the Moon's true Right Ascension for that Time. The Moon's Right Ascension and Declination, are as they would appear if they could be observed from the Centre of the Earth.

3. The Sun's observed Altitude must be cleared from Semidiameter, Dip, and Refraction, to get his true Altitude. Likewise,

4. The Moon's observed Altitude, must be cleared from Semidiameter, Dip, Refraction and Parallax in Altitude to get her true Altitude. The Semidiameter is in the Ephemeris. The Dip is in the Table for Dip of Horizon, and is always subductive. The Refraction is in its Table, and is subductive. The Parallax in Altitude, is in its Table, and is additive.

5. Having the Sun's observed Altitude thus cleared, the Moon's observed Altitude thus cleared, and the Equatorial Distance of their Centres; assume some Latitude at Discretion, and with these four Data, make one or more Operations (as in the foregoing Section) and the true Latitude will be thereby found.

6. When a Ship at Sea has been driven by stormy and tempestuous Weather, Observations of this kind made when either of the Luminaries is not far from the Meridian, may be applied; and not only the Latitude, but likewise the Longitude found, after an easy Process. The Operations for the Latitude have been exemplified in the former Section, those for the Longitude as in Sections 104, 108, 109, 110, 111, 112. When the Latitude is known somewhat near the Truth, the true Latitude will be soon approximated; and then the Longitude may be calculated.

7. The Difficulties attending the Solution of this Problem by the direct Methods of proceeding to an Answer, will ever render it next to insuperable; whilst this Method may be practised with Ease and Success, by Persons who are unacquainted with the Properties of the Sphere and spherical Triangles. When the Moon is the nearest Luminary to the Meridian or farthest from it, the Effect of her Parallax in Altitude may be taken out by Inspection, as readily as the other Effects on her Altitude; and when the Sun is near the Meridian, it will appear by his slow Increase or Decrease of Elevation.

8. This is a Method of taking the Longitude at Sea, purely astronomical, depending on its own Principles, and no way apt to be erroneous through the Imperfections of preceding Observations or intermediate Measures of Time, Distances, or other Defects.

9. The first Solution of this Problem was given by me in the two following Examples.

EXAMPLE I.

In a certain Year, near the Beginning of a given Month, in an unknown South Latitude, the Sun being near the Meridian of the Place of Observation; the Altitude of the Sun's Centre was taken 59° 12′, the Altitude of the Moon's Centre taken 27° 3′, and the Distance of Sun and Moon's nearest Limbs taken 58° 52′ 48″, all at the same time. The Latitude and Longitude of the Place of Observation, are found thus.

1st. By the rough central Distance, the Day and Hour, were, December 7th at 7 Hours.

2d Hereby and the Ephemeris, the Moon's Right Ascension was 316° 26′, and her Declination 14° 35′ S. The Sun's Right Ascension was 254° 37′ and Declination 22° 43′ S. The Equatorial Distance of Sun and Moon was 61° 49′, the Moon East of the Sun.

3. By assuming and approximating after the foregoing Method, the Latitude was 53° 28′ S. and the Solar Time at the Ship was 0h 6′ 12″ short of Noon.

4th. The true Distance of Centres cleared from Refraction and Parallax was 58° 43′ 10″. The Solar Time at Greenwich was 6h 56° 14″ past Noon. Hence, the Ship's Longitude was 105° 36′ West of Greenwich.

EXAMPLE II.

In a certain Year, about the Middle of a given Month, in an unknown South Latitude, Regulus the Lion's Heart being nearest to the Meridian, west of it and the Moon west of Regulus; the Altitude of Regulus was taken 20° 8′, the Altitude of the Moon's Centre taken 18° 56′, and (the Moon appearing near the Full) Regulus's Distance from the Moon's farthest Limb was taken 28° 14′ 39″, all at the same time. The Latitude and

and Longitude of the Place of Observation, are found thus.

1st. By the rough central Distance, the Day and Hour were, February 17th, at 10 Hours.

2d. Hereby and the Ephemeris, the Moon's Right Ascension was 176° 1', and her Declination 3° 48' N. Regulus's Right Ascension was 148° 59', and Declination 13° 4' N.

3d. By assuming and approximating after the foregoing Method, the Latitude was 54° 47' S. and Regulus was near Twenty Degrees East of the Meridian of the Place of Observation.

4th. The true Distance of Centres cleared from Refraction and Parallax was 28° 9' 8". The Solar Time at Greenwich was 9^h 49' 11" short of Noon, and at the Ship 10^h 30' 44". Hence, the Ship's Longitude was 10° 23' East of Greenwich.

10. The Lunar Theory is already brought to a greater Degree of Perfection, than the generality of Persons who make long Voyages are aware of. Instruments sufficiently correct for taking the angular Distances may be had of able Instrument Makers. Almost every Person hath Judgment and Agility enough to take the cotemporary Observation, whether it be the Altitude of the Sun, Star, or Moon, or the angular Distances. The Opportunities are numerous. The Success of the Lunar Method depends wholly on its own Principles. The cotemporary Observations can be made with Accuracy, and the Longitude may be taken by it without other Aids, whether one of the Luminaries be near the Meridian or remote from it.

CXXVI.

Having either, the Altitude of the Moon not far from the Meridian, and the Altitude of a Zodiacal Star any how situated; or the Altitude of a Zodiacal Star when not far from the Meridian, and the Altitude of the Moon any how situated; also, the angular Distance of Sun and Moon's Limbs, all taken at the same time; to find the Latitude and Longitude of the Place of Observation.

1. Here *1st.* The Solar Time at the Ephemeris's Place is to be found. *2d.* The Solar Time at the Ship is to be found. *3d.* The Sum or Difference of these two Times is the Longitude. This Problem has been illustrated, in Example 2. of the former Section.

2. If the Observer doth not choose to infer the Ship's Time from the Zodiacal Star and Moon, when the Star is nearest to the Meridian of the Place of Observation, he may take the Altitude of any other Fixed Star, or a Primary Planet, and use it instead of the Moon to get the Ship's Time. Or,

4. He may use any other Stars (one of which is nearer the Meridian) for the same Purpose.

CXXVII.

Having an Altitude of the Sun, an Altitude of the Moon and the Distance of Sun and Moon's Limbs, all taken at the same time, and when neither of the Luminaries is near the Meridian of the Place of Observation, also, an Ephemeris; to find the Latitude and Longitude of the Place of Observation.

1. In order to solve this problem, first, the Altitudes must be cleared from Semidiameter, Dip, and Refraction, Parallax in Altitude, and the Equatorial Distance of the Centres must be found, as in the former Problem; this prepares the Data for Operation as follows.

2. Assume any two Latitudes as near the Truth as may be reasonably supposed from the Situations of the Sun and Moon at the time of Observation; with these, the Co-altitudes, and the Equatorial Distance, compute in the same manner as was directed for the Latitude in Section 121, having one Latitude by Account, another assumed, two Altitudes, and the Elapsed Time, until the true Latitude is known.

3. Having the Latitude and the three cotemporary Observations; first, from the Sun's Altitude the Moon's Altitude and Distance of their Limbs, the Time at the Ephemeris's Place is to be found (as in Section 109 or 110), and secondly, having the Latitude, Sun's Altitude, and Polar-distance, the Time at the Place of Observation, either past or short of Noon.

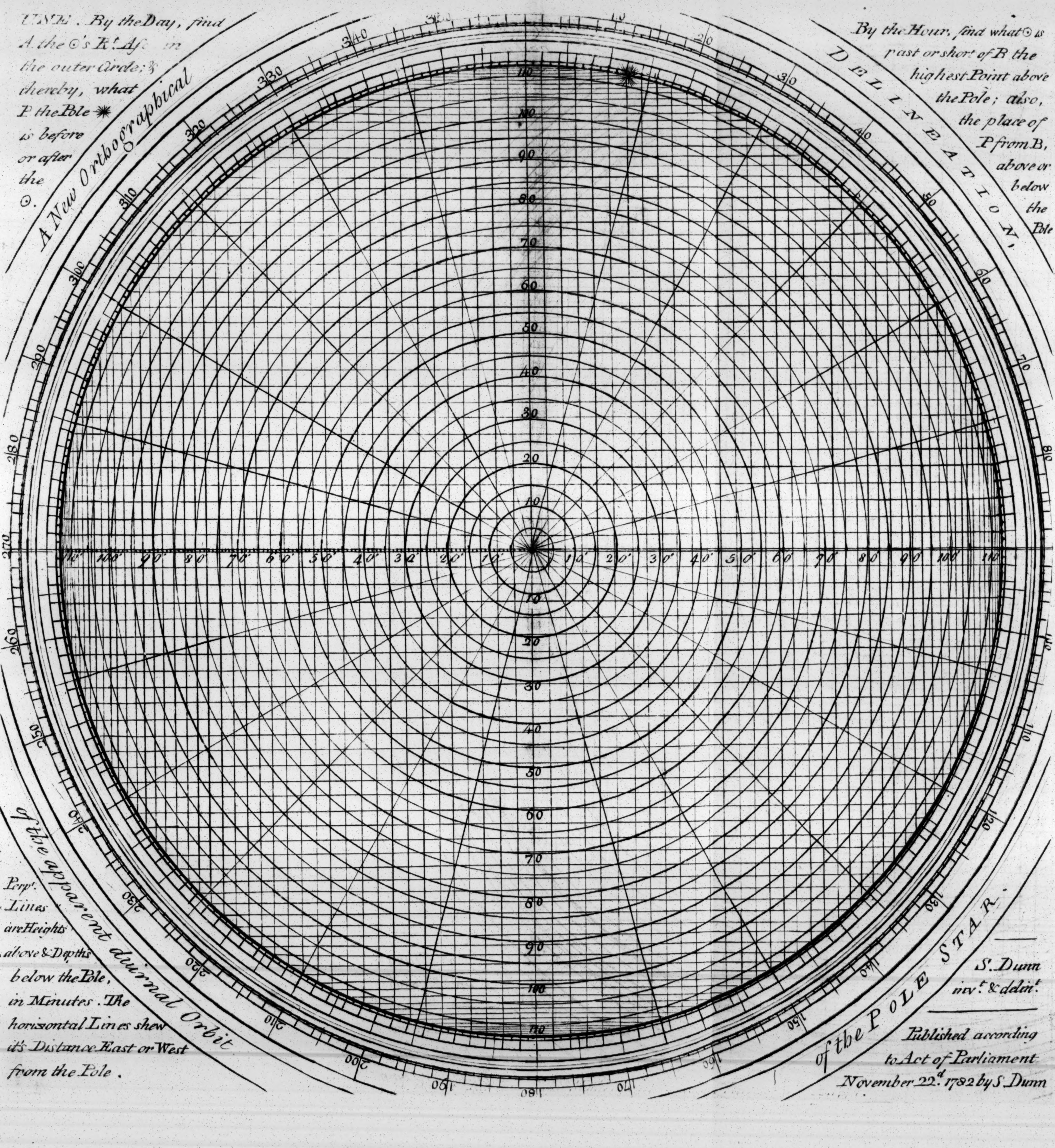
4. The Sum or Difference of the two Times, is the Longitude, from the Ephemeris's Place, as in Section 112.

CXXVIII.

Having the Altitude of a Zodiacal Star, situated eastwardly, westwardly or under any Position not near the Meridian, the Star's Altitude, Moon's Altitude, and Distance of Star and Moon's nearest or farthest Limb, at the same time, and an Ephemeris; to find the Latitude and Longitude of the Place of Observation.

1. In order to solve this Problem; first, the Latitude must be found by the Star and Moon, as in the former Problem by Sun and Moon; secondly, by the Star's Altitude, Moon's Altitude, and Distance, the Time at the Ephemeris's Place must be found; thirdly, the Sum or Difference of the two Times, is the Longitude required.

This Problem may be practised in most Parts of the Torrid Zone with Ease, because there the Moon and Star will often be near the same Vertical Circle, and the Horizon will be illuminated by the Moon.


A New Orthographical DELINEATION, of the apparent diurnal Orbit of the POLE STAR.
USE. By the Day, find A the ☉'s Rt. Asc. in the outer Circle; & thereby, what P the Pole ✱ is before or after the ☉.
By the Hour, find what ☉ is past or short of B the highest Point above the Pole; also, the place of P from B, above or below the Pole
Perpr. Lines are Heights above & Depths below the Pole, in Minutes. The horisontal Lines shew it's Distance East or West from the Pole.
S. Dunn inv.t & delin.t
Published according to Act of Parliament November 22d. 1782 by S. Dunn

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CXXIX.

Having any two Altitudes of either the Sun and Moon or the Moon and a Zodiacal Star, alſo, an Ephemeris; to find the Latitude and Longitude of the Place of Obſervation.

1. At the time of making the cotemporary Obſervations, the poſitions of the Sun and Moon to each other in the Day, and of the Moon and Stars in the Night, will, in many Caſes, indicate the Zone in which the Obſerver is, this is all that is wanted previous to a Computation from theſe Data only.

2. Hence alſo it will appear whether the Luminaries or either of them, are near the Meridian or remote from it. If either of them is ſuppoſed near the Meridian, the Method in Section 125 is to be applied for the Sun and Moon, and that of 126 for the Moon and Star. If both Luminaries are ſuppoſed remote from the Meridian, the Method in Section 127 is to be applied for the Sun and Moon, and that of 128 for the Moon and Star.

3. If there is any reaſon to doubt of Convergency in either of theſe Methods, both of them may be applied, by the ſame Data, and the anſwer ariſing from both will be a Confirmation.

4. Although this Problem may be ſolved without any previous knowledge of the Latitude, if that is wanted within a few Degrees of the Truth, the minutely Aſcent or Deſcent of the Sun in the Day, and the Poſitions of the circumpolar Stars in the Night are ſufficient to ſhew it; and then, the Problem may be ſolved after the ſame manner as though there were given an erroneous Latitude by Account and the other Data.

CXXX.

Of Latitude by having the Altitude of the Pole Star, either on the Meridian or out of the Meridian.

1. By comparing together the Obſervations made of the Pole-ſtar's Diſtance from the true Pole of the Equator for One hundred and eighty Years paſt (from my Practical Aſtronomy) the Pole-ſtar's Diſtance from the Pole will be a Degree and three quarters in the Year Eighteen hundred and one; before and after that time its Approach toward the Pole will be a Minute of a Degree in three Years, its Diſtance from the Pole in Seventeen hundred eighty-ſeven being a Degree and fifty Minutes.

2. The Pole-ſtar is between the ſecond and third Magnitude, of a red colour and ſparkles leſs than ſome other Fixed Stars of its Magnitude. If this Star had the Magnitude and Splendour of Sirius, it might be applied with great Eaſe and Succeſs for determining the Latitude, in the Night, at many places on Land and at Sea, in North Latitude, as well out of, as on the Meridian.

3. In order to take the Altitude of this Star, the Obſerver ſhould be provided with a Sextant or Octant, having a ſmall Teleſcope magnifying two or three times at the Place of the Eye, the Obſervations ſhould be made when the Horizon is beſt defined and the Image of the Star beſt to be ſeen on the quickſilvered Part of the ſmall Mirrour.

4. At Sea, the Time of the Night when the Star is on the Meridian either below or above the Pole, is known by comparing its Right Aſcenſion, with that of the Sun and the Hour at the Ship; then, the Star's Altitude when below the Pole, added to its Polar-diſtance, gives the Latitude; but, when the Star is above the Pole, its Polar-diſtance ſubtracted from its Altitude gives the Latitude.

5. In either of theſe Caſes or any other, the obſerved Altitude of a Star is to be firſt cleared from Dip and Refraction, by ſubtracting both; the Remainder is the Star's Altitude, from which the Latitude is to be inferred.

6. When the Pole-ſtar is not on the Meridian of the Place of Obſervation, its Poſition to the Pointers or ſome other Stars near the Pole, is to be carefully noted by the Obſerver, and (by the Poſition of the Pointers) the Pole-ſtar's Elevation or Depreſſion above or below the horizontal Line paſſing through the true Pole. By this Poſition, find the Place of the Star in the Circumference of the Orthographical Delineation of its Orbit, and the Elevation above or Depreſſion below the true Pole will be ſhewn by Inſpection. See the Diagrams.

CXXXI.

Of the Terreſtial Globe; its Uſes and Defects at Sea.

1. The Surface of the Terreſtial Globe, repreſents (in Miniature) the Surface of the Earth and Seas; and when it is made a Foot in Diameter, the Length of a Degree of Latitude on its Surface, is but little more than the Tenth of an Inch, Engliſh Meaſure.

2. The principal Problems of the Terreſtrial Globe, are theſe, *1ſt.* To find the Diſtance of any two Places. Meaſure it with the Quadrant of Altitude, or a Pair of Compaſſes, or a Slip of Paper, reckoning Sixty nautical Miles, or Sixty nine and half Engliſh Miles to a Degree of Diſtance. *2d.* To find the Latitude of a Place. Bring the Place to the Braſs Meridian, and the Latitude, whether it be north or ſouth, is the neareſt Diſtance from the Place to the Equinoctial

in Degrees and Minutes. *3d.* To find the Longitude of a Place. Bring the Place to the Brass Meridian, and the Longitude from London, is the Distance on the Equinoctial, from the Meridian of London to the Meridian of the Place. *4th.* To find the Places where the Sun is Vertical at Noon, on a given Day. Find how much the Sun is northward or southward of the Equinoctial at Noon, by the Scale of Declination, and all the Places at that Distance from the Equinoctial are the Places required. *5th.* To find where the Sun is Vertical at a given Hour of a given Day. Find the Distance of the Sun from the Equinoctial, north or south thereof; then, (reckoning fifteen Degrees to an Hour) count as many Hours to the Eastward or Westward on the Equinoctial, as is the given Hour, and in that Meridian, at the forementioned Distance north or south of the Equinoctial, is the Place required. *6th* When the illuminated Hemisphere, is to be shewn for a given Hour. Find the Place of the Earth or Sea where the Sun is vertical; bring this uppermost and the illuminated Hemisphere is above the Horizon. *7th.* When a Place is brought uppermost, the Bearings of other Places therefrom are shewn on the Horizon of the Globe.

3. However large this Globe is made, the Difficulties of making it truly, and the Errors arising in its Materials and Use, render it at best but an ingenious Model for giving Ideas concerning such things, as are to be more accurately found for Use at Sea, by the Application of accurate Instruments and proper Computations.

CXXXII.

Of the Celestial Globe; its Uses and Defects at Sea.

1. The Surface of the Celestial Globe, represents the Concavity of the Celestial Epanse or Starry Heavens, in which Surface the Sun, Moon, Planets, and Fixed Stars, appear as though they were placed, at equal Distances from the Eye of a Spectator on the Earth's Surface.

2. When the Globe is made a Foot in Diameter, a Degree of Distance on it is but little more than a Tenth of an Inch, the smallness of which Measure (however accurately the Globe is made) will not admit of those nice Subdivisions which Astronomical Observations require. There is another Circumstance attending the Delineations on this Globe; the Fixed Stars have their Places in Longitude on it for some particular Year, and as the Stars have a Precession in Longitude by reason of the Recession of the Equinoctial Points, in the Interval of Seventy one Years and half; in that time, the Places of the Stars become different in the Heavens a Degree of Longitude, and in proportion for a less number of Years.

3. The Principal Problems of the Celestial Globe are these. *1st.* To set the Globe to the Latitude of a Place. Elevate the Pole of the Globe above the Horizon, as much as is the given Latitude. *2d.* To set the Globe to the Latitude of a Place, and to the Sun's Place in the Ecliptic. Elevate the Pole to the Latitude of the Place; then bring the Sun's Place in the Ecliptic for that Day to the Brass Meridian turn the Hour Index to the Meridian and then the Globe is ready for solving the following Problems. *3d.* Turn the Sun's Place eighteen Degrees below the Horizon, East and West, the Hour Index will shew the Beginning and End of Twilight. *4th.* East and West in the Horizon, it shews the Time of Sunrising and Sun-setting. *5th.* At the same Places, the Amplitude from true East and true West. *6th.* Over the East and West Points, it shews the Hour and Altitude. *7th.* Turn the Sun's Place to any Altitude, the Hour is shewn. *8th.* At any Hour the Altitude is shewn. *9th, 10th.* At any Hour or Altitude the distance from the point under the Sun's Place to that under the elevated Pole, is the Azimuth. *11th. 12th.* Turn the Globe to any given Altitude of Sun or Star the Hour and Azimuth are shewn. *13th. 14th.* Turn it to a given Azimuth, the Hour and Altitude are shewn. *15th. 16th.* Turn the Globe to a given Hour, the Stars on the East and West Side of the Meridian continued quite round, are shewn for that Time. *17th.* The Places of the Moon and Primary Planets, being found either in or near the Ecliptic, the same Problems are solved concerning them as for the Fixed Stars. *18th.* The circumpolar Stars that do not set; the Parts of the Equator, Ecliptic, and other Circles of the Heavens, are hereby represented in Miniature, as they are in the Heavens at their respective Times.

CXXXIII.

The Description and Use of the Charts of Zodiacal Stars; designed for facilitating the Lunar Method of finding the Longitude at Sea.

1. A Chart of the Zodiacal Stars, is a Delineation of the Zodiac, the Circles adjacent, the Fixed Stars within the Zodiac, those that are adjacent thereto, and such other Fixed Stars as may be applied in Conjunction with the Zodiacal Stars, in the Lunar Method of finding the Longitude.

2. This Delineation is of two kinds; the first is for shewing the Fixed Stars and the Circles of the Heavens as they appear in the Concavity of the Heavens; the second kind is for shewing those

Stars

Stars and Circles as they would appear, if they were impressed down on the Horizons of Places quite round the Earth's Globe; or otherwise, as delineated to face the Heavens. The former is called the first Form; the latter, the second Form of these Charts.

3. In both of these Charts, the Equator is a straight Line drawn through the Chart, divided into three hundred and sixty Degrees, also into Twenty four Hours of Time. The Ecliptic cuts the Equator at the Beginning and Middle thereof, this Line is the Sun's Path, and is divided into its twelve Signs with Subdivisions. The Zodiac is the Zone or Breadth on each Side of the Ecliptic, in which the Moon and Primary Planets are always, somewhere or other to be found.

4. The Sun's apparent Path is from Aries to Taurus, eastward, in the Ecliptic Line at the rate of a Degree per Day nearly. The Moon's apparent Path is the same way, but quicker and out of the Ecliptic Line in the manner before decribed in this Work. In this Zodiac, the Primary Planets have their apparent Stations, direct and retrograde Motions.

5. At the Time of the Moon's Change, (commonly called New Moon) the Moon's Longitude is the same as that of the Sun, although the Moon may have north or south Latitude. After the Change, the Moon recedes eastward in the Zodiac with a swift Recession (before described) first appearing like a Crescent or horned, next gibbous or more than half full, and then full, near that Point of the Zodiac which is opposite to the Sun; during this Interval the illuminated Part is toward the Sun from which she has apparently departed. The remaining part of the Lunar Month, the Moon is losing her Illumination on her western Side, retaining it on her eastern Side, and approaching by her daily Recession, toward the western Limb of the Sun till the next Change, or New Moon.

6. Having the predicted Longitude and Latitude of the Moon for each Three Hours or any other small Intervals of Time throughout the Lunar Month, her Place may be found in Miniature on this Chart, and the Position of her Cusps or Corners, except at the full; likewise the Position of the Line joining the Cusps to the nearest and remote Zodiacal Stars. The Line at Right Angles to the Cusps, commonly makes an Angle of not more than five Degrees with the Ecliptic. This and the predicted Distances from the nearer Zodiacal Stars at the same time, makes them easily to be found, either by the Eye or a Sweep with Hadley's Instrument. By this Method the Zodiacal Stars may be easily found.

CXXXIV.

Problems on the Charts of Zodiacal Stars; and the manner of applying them in the Lunar Method.

1. Having Sights of the Sun and Moon; to determine the Position of the Ecliptic and Zodiac, at that Time.

Suppose a straight Line to join the two Cusps or Corners of the Moon; bisect this Line at right Angles with the Arch of a great Circle passing one way toward the Sun and the other way from the Sun, this will be nearly the Position of the Ecliptic, which was required.

2. Having Sights of the Moon and a Zodiacal Star in the Night; to determine the Position of the Ecliptic and Zodiac, at that Time.

As before, bisect the Line joining the Moon's Cusps, and continue it both ways, and it will be nearly the Position of the Ecliptic and Zodiac, which was required.

3. Having the Latitude of a Place; to find its Parallel of Declination on the Chart of Zodiacal Stars.

Guide your Eye north or south of the Equator, on the Chart of Zodiacal Stars, 'till you come to the given Latitude; and the Line at right and left, keeping at an equal Distance from the Equator, is that which was required.

4. Having the Latitude of a Place, the Day of the Month and Hour of the Day or Night; to find the Zenith Point on the Chart of Stars, and the Stars near it at that Time.

Find the Parallel of Declination for the given Latitude, as in the former Problem; then, guide your eye along that Parallel to the right or left, until it is as many Hours and Minutes either past Noon or short of Noon, as is the given Time; and it gives the Point required. The Stars near to this Point are near the Zenith, the Line drawn through it north and south is the Meridian, and the Stars near it are near the Meridian of the Place at the given Time. The Stars East of this Sine are short of the Meridian, the Stars West of it are past the Meridian, and so many Degrees or Hours and Minutes, as are shewn at the Equator.

5. Having the Latitude of a Place, the Day of the Month, and Hour of the Night; to form an Estimate, what may be the Positions of the principal Fixed Stars, toward the Zenith and Equator, at that Time.

Find the Place of the Zenith as in the two former Problems; and it will be shewn, not only what Stars are near the Zenith and Equator at that Time, but also what Parts of the Ecliptic and Zodiac are near the Meridian east and west, and how the Zodiacal Stars or other Fixed Stars not far from the Zenith and toward the Equator, are situated at the same Time.

6. Having the Latitude of a Place, the Day of the Month, and Hour of the Night, with an Ephemeris; to find the Situations and Positions of the Moon and Primary Planets, to the Zenith and Meridian at that Time.

Find the geocentric Latitudes and Longitudes of the Moon and Primary Planets in the Ephemeris, and mark those Places in the Zodiac of the Chart. Then, find the Zenith Point and Meridian for the given Day and Hour, and the Positions of the Moon and Primary Planets to the Zenith and Meridian, will appear by Inspection.

7. Having the Day of the Month, and the Latitude of a Place, to judge nearly what may be the Sun's Right Ascension and Declination, and how near the Sun may apparently approach toward the Zenith, on that Day at Noon.

By the Day of the Month, find the Sun's Longitude in the Ecliptic, then, his nearest Distance from the Equator is the Declination, and where his Meridian comes to the Equator, is the Right Ascension; the Distance from the Sun's place in the Ecliptic to the Zenith Point, is the meridional Zenith-distance; which were required.

8. To estimate nearly what may be the Right Ascensions, Declinations, Longitudes and Latitudes of the Fixed Stars near the Ecliptic; by the Charts of Stars.

The Right Ascensions are had by referring them along their Meridians to the Equator, on which their Right Ascensions are reckoned; their Declinations are their Distances from the Equator; their Longitudes are referred (by perpendicular Arches) to the Ecliptic, on which their Longitudes are reckoned; and their Latitudes are their Distances from the Ecliptic, either north or south thereof.

9. Having the Day of the Month, the Hour of the Night, the Latitude of the Place, and an Ephemeris; to find in what Parts of the Heavens the Primary Planets and Zodiacal Stars are, and how they are situated to the Moon at that Time.

Find the Moon's Place by the Ephemeris for the given Time, also the Places of the Primary Planets, and mark those Places in the Zodiac of the Chart of Stars; then find the Zenith Point for the Time of Observation, and its Meridian, so the Zodiacal Stars, the Primary Planets and the Moon, will appear in Miniature on the Chart, situated to each other (either above or below the Horizon) as they are at that time in the Heavens.

10. To distinguish the Primary Planets, from the Zodiacal Stars, or any other Fixed Stars, appearing near the Moon's Path in the Heavens.

When the Moon has her Cusps or Corners, the Line drawn at Right Angles to that of her Cusps, will shew whether a Fixed Star is near the Moon's Path or not, by this the Observer may often be undeceived; if this is not satisfactory, note as follows. The Planet Mercury is very seldom remote enough from the Sun to cause a Mistake. Venus and Jupiter almost always appear very large. Mars and Saturn at their greatest Distance, appear small; and, at all great Altitudes the Primary Planets never twinkle, these Properties with their Colours (before described in Section 24) and their being near the Ecliptic Line, will point them out, to be avoided. Here, the Observer should never forget that, all the Fixed Stars twinkle, except when they are near the Zenith, then their Scintillation or Twinkling ceases, but the Planets never twinkle when they are elevated but a few Degrees. Near the Horizon, the larger Planets may have a small Scintillation. Mercury appears small, of a pale Colour, and is never far from the Sun. Venus appears frequently large and splendid to the naked eye, when she appears but half or a third part full through a Telescope. Mars changes his apparent Magnitude more than any other Primary Planet, but always keeps a red Colour; at his greatest Distance, he is not larger than Regulus, at his nearest Distance, as large as Jupiter when farthest from the Earth. Jupiter commonly appears large and refulgent. Saturn commonly appears pale, sometimes like a Star of the first, at other times of the second Magnitude. These Changes in their apparent Magnitudes, arise from their Change of Distance from the Earth, or their Places in their Orbits compared with the Place of the Earth in its Orbit.

11. Having the Time at the Ephemeris's Place, and the Time that a Zodiacal Star is either past the Meridian or short of it at the Ship's Place; to determine the Solar Time at the Ship's Place, and by both of these the Ship's Longitude.

In this Problem, the Charts of Zodiacal Stars are of ready Use for removing Difficulties that arise with Persons who have but an imperfect Idea of the Situations of the Celestial Bodies, and their apparent Motions in the Circles of the Celestial Sphere.

In the Lunar Method, by the Moon and a Zodiacal Star; when the Time at the Ephemeris's Place is found, likewise the Star's equatorial Distance either past the Meridian of the Place of Observation or short of it, the Solar Time at the Place of Observation may be inferred thus.

1st. Subtract the Right Ascension of the Sun from the Right Ascension of the Star, or the Right Ascension of the Star from that of the Sun (which of the two can be done) the Remainder is their Equatorial Distance.

2*d*. When the Star is short of (or eastward from) the Meridian of the Place of Observation, remove that Meridian so much westward from the Star, as the Star is eastward from the Meridian. When the Star is past (or westward from) the Meridian of the Place of Observation, remove that Meridian so much eastward from the Star, as the Star is westward from the Meridian Then,

3*d*. At the Equator on the Chart, are three Points, different in Right Ascension (or Distance from the Beginning of the Equator) namely, the Right Ascension of the Sun, that of the Star, and that of the Meridian. Of these, the Sun's Right Ascension is accurately known by the Ephemeris's Time; the Star's Right Ascension, is known from its Table; the Star's equatorial Distance from the Meridian; either past the Meridian or short of it, is known from the Star's observed Altitude, Polar-distance, the Latitude, and a Computation of that Distance. Consequently,

4*th*. By adding or subtracting (as the Case requires) the Sun's equatorial Distance from the Meridian is had, and this is the Solar Time at the Ship, either past or short of Noon. Then, the Sum or Difference of the Ephemeris's Time and Ship's Time, is the Longitude.

Here, it is evident that, if the Star's Altitude is correctly taken, the Ship's Time is right, and that, an Error in that Altitude produces a proportional one in the Longitude.

CXXXV.

Of the general Methods, which have been successfully applied, for finding the Longitude at Sea.

1. Although there have been Twenty Methods proposed for finding the Longitude of a Ship at Sea, there have been but five of them improved enough to answer their Design. The first of these Methods is, by the Ship's Reckoning; the second, by help of a Watch or Time-keeper; the third, by Sun and Moon, or Moon and Zodiacal Stars; the fourth, by help of Variation Charts; the fifth, by Calculations and Observations, relative to the Immersions, Emersions, Occultations and other Phænomena of Jupiter's Satellites.

2. The fifth Method by the Satellites has been practised by Help of a Marine Chair to take off the Ship's Motion, and at certain times with Success, but this Apparatus is thought too unweildy for general use, though rendered much more convenient and complete than it would otherwise be, when furnished with the most improved Telescopes.

3. Whenever this Method by Jupiter's Satellites is brought into use at Sea, it will have many Imperfections and Obstructions attending it, which Persons who are advocates for it may not think to exist. 1*st*. Whatever Errors are in the predicted Times of the Satellites appearing or disappearing, will be Errors in the Longitude inferred. 2*d*. The Telescope must be applicable at Sea, at Times and Places when and where the Longitude is wanted to be known. 3*d*. The Observer must be acute and expert enough to make just Allowances for all Changes of the Air and Vapours, where and whenever the Observations are made; and be undeceived in judging which is the Satellite to be observed, before the Immersion, or to be noted after Emersion 4*th*. He must know, not only the Ship's time, but also the Ship's Longitude, somewhat near the Truth; otherwise, his attention must be exerted in a strict manner for such length of Time as will quite weary him. 5*th*. The Planet Jupiter is not always visible in the Night, even in the most favourable Weather, on account of his frequent Proximity to the Sun. 6*th*. As to the positions and near Approaches of the Satellites to each other; these, at Sea, will have greater Difficulties, in attending Observations of them.

4. It remains therefore, to take the Longitude at Sea, by the four other Methods before mentioned.

CXXXVI.

Of the four principal Methods, which have been successfully applied, in finding the Longitude at Sea.

1 In Order to form a judgement concerning the Merit of each of the four Methods whereby the Longitude has been found, it is proper to examine, how frequently and to what Accuracy, each of them may be practised, when and where they are wanted at Sea; because, from these Circumstances their Utility will arise, and it will thereby appear how much Navigation is assisted by each of them.

Advantages of the first Method; by the Ship's Reckoning or usual Method of Sailing.

1*st*. This Method, the Navigator must be acquainted with, whatever other Method he thinks proper to practice at any particular time or place; or whether he is acquainted with any other or not. 2*d*. This Method is easy, and when the Ship makes a short Voyage, the Error in Longitude by it may be expected very small; perhaps, when practised with Care and the latest Improvements, in short Voyages, it is the most certain Method that can be used. 3*d*. It is universal, may be applied at all times, as well by Night as by Day, and a Voyage is never hindered in it for want of seeing the Sun. 4*th*. It is the only Method whereby the direct Course of a Ship can be known, to sail

the ſhorteſt Way to the intended Port. *5th.* In it, by help of the obſerved Latitude, the ill Effects of Currents and Leeway may often be diſcovered and allowed for, and thereby the Longitude determined near the Truth. *6th.* It is the only Method whereby the Variation, Leeway, Latitudes, Longitudes, and other Incidents, can be examined throughout a ſmall Part of an Ocean or Sea, and theſe are principal Things, in Practical Navigation.

Diſadvantages of the Firſt Method; by the Ship's Reckoning or uſual Method of Sailing.

1ſt. It is liable to great Error at the End of a long Voyage, and ſometimes in a ſhort Interval ſmall Errors will creep into the Reckoning and cauſe fatal Accidents. *2d.* Whatever Errors ariſe in this Method, they cannot be corrected by any Principles or Practice belonging to this Method. Thoſe Errors are carried on in the Ship's Reckoning, and Corrections of them muſt ariſe from Principles no way related to this Method. *3d.* When the Ship comes in Sight of Land, it may or it may not be Land with which her Navigators are acquainted with, and a Miſtake in this reſpect may be attended with fatal Conſequences. *4th.* Through not knowing the Longitude, the Ship, may be in a Track very different from that ſhe is ſuppoſed to be in, and may thereby come near Rocks or Shoals in the open Seas, which ſhould be avoided.

Advantages of the ſecond Method; by the uſe of Watches or Timekeepers.

1ſt. This Method has the appearance of being the eaſieſt to be practiſed, and with all deſireable Accuracy when the Latitude is known, and the Watch or Timekeeper keeps correctly to mean Solar Time; then, the Navigator is carefully to obſerve for the Latitude as correctly and often as poſſible, ſo that it may be always ready for Uſe in this Method; and let the Timekeeper go on undiſturbed. *2d.* An Altitude of either Sun or Star, the Declination, and the Latitude, are all that are wanted to give the Solar Time at the Ship; and this compared with the Solar Time ſhewn by the Watch, gives the Longitude.

Diſadvantages of the ſecond Method; by the Uſe of a Watch or Timekeeper.

1ſt. This Machine muſt be curiouſly conſtructed and finiſhed in all the Parts that are deſigned for making it keep Equal Time; this requires much Ingenuity, Skill and Care in an Artiſt, and therefore he will be entitled to ſuch a Price for his Workmanſhip as it deſerves. *2d.* That Price may be unſuitable with many Perſons, who want to find the Longitude at Sea, for the Safety of themſelves, and of the Property of their Employers. *3d.* The Machine muſt have a ſtrict Trial before it is taken to Sea; and it can no way be effected completely, but by the moſt accurate aſtronomical Obſervations. *4th.* If an Accident happens to it, or otherwiſe it errs in its uniform Rate of going, at Sea, there is nothing belonging to it that can ſhew what the Error is, or Point out a Correction. *5th.* Such Machines may go well for a conſiderable Interval, or for but a very ſhort Time, and then alter, through Cauſes or Accidents unforeſeen, unknown, and no way to be accounted for.

Advantages of the third Method; by the Sun and Moon, or Moon and Zodiacal Stars.

1ſt. This Method may be practiſed in the Day time almoſt half the Days in the Year, with no better Inſtruments that ought to be in every Ship for finding the Latitude, and with which Navigators are well acquainted. *2d.* Almoſt all the Year (by Day and Night) it may be practiſed with Inſtruments to be purchaſed at an eaſy Price, and by Calculations no way difficult to be learnt, nor much longer in Operation than the Method by a Timekeeper. *3d.* The Longitude may be taken by this Method, at different times in the Day, alſo at different times in the Night, alſo by a Star Eaſt and another Weſt of the Moon, all independent of each other, and the Medium of the whole may be taken. Such Operations may be continued, from Obſervations made Day after Day, all independent of each other; this no other Method admits of. *4th.* In this Method the cotemporary Obſervations only, are ſufficient Data for the eaſy Determination of both the Latitude and Longitude of the Ship, although the former be unknown, and the Operations may be made by Perſons of no previous Knowledge in the Theory of Aſtronomy.

Diſadvantages of the third Method; by the Sun and Moon, or Moon and Zodiacal Stars.

1ſt. During three or four Days in each Month, the Moon will be ſo near to the Sun, that Obſervations cannot be made in this Method; and therefore, before ſuch Intervals begin it will always be proper to omit no Obſervations that can be made in this Method, to the Beginning of that Interval. In the Interval the firſt Method is to be carefully applied, and (if the Navigation requires it) the Watch may alſo be applied, to get a Medium from both.

Advantages of the fourth Method; by the Help of Variation Charts of the Magnetic Needle.

1ſt. This Method requires no other Inſtruments, Books nor Inſtructions, than what Navigators are

or

or ought to be acquainted with, in the Practice of the first Method. *2d*. When the Ship is in a Latitude that is nearly known, and at a Place in an Ocean or Sea, where it is known that the Lines of equal Variation do run northwardly and southwardly, and that the Variation in those Lines alters much in small Distances, after a regular manner; the Longitude is taken easily by this Method, thus. *3d*. Having the Latitude either by Observation or Account, the Altitude of either Sun or Star, and its Polar-distance, the true Azimuth is found; which being compared with the Magnetic Azimuth gives the Variation. *4th*. This Method is not dependent on any previous Knowledge of other Data than the Latitude when the Star is observed, nor of any thing but the Latitude and Sun's Declination when the Sun is observed. *5th*. The Moon may also be applied in this Problem, and Observations may be repeated by different Fixed Stars. *6th*. In the midst of large Oceans and Seas, where Ships commonly lose their Longitude, it may be taken by this Method, and applied in sailing the Course and Distance discovered to the designed Port.

Disadvantages of the fourth Method; by the help of Variation Charts of the Magnetic Needle.

1st. The Lines of equal Variation, do, at some Places, run such Lengths, near the same Parallels of Latitude, that the Longitude cannot be found by this Method at such Places. *2d*. At some other Places where the Lines of equal Variation do run northwardly and southwardly, they are too wide for the Longitude. But, *3d*. The Places where these Positions of the Variation Lines are to be met with, are but few being compared with all others which ships navigate in important Voyages.

CXXXVII.

Of the Accuracy to which the Longitude may be taken by the usual Method of Sailing, the Use of Watches or Time-keepers, the Sun and Moon or Moon and Zodiacal Stars, and by Variation Charts of the Magnetic Needle.

First Method; by Sailing.

1. However perfect and useful this Method may be thought for short Voyages, it sometimes happens that in long Voyages, Navigators are deceived by it several Hundreds of Miles. Some Navigators will have it, that by a due Correction of the usual Method of Sailing, a Ship's Reckoning will (in crossing the Atlantic Ocean under one direction) not differ from the true Longitude more than a few Miles. Others (the most experienced Navigators) in sailing from South to North homeward bound, have lost their Longitude by Reckoning, from a few to more than Five hundred Miles. Such Errors as these are of great Consequence, and must arise from some Cause or Causes, for which the usual Method of Sailing has no Correction. It is a well received Opinion, that the Oceans and open Seas have, at certain times strong Currents, setting Ships under unexpected Directions, and such as can never be accurately predicted nor estimated.

Second Method; by Watches.

1. The Accuracy to which the Longitude can be taken by this Method depends on the Perfection of the Watch or Time-keeper that is to be used, and the length of Time the Ship is on her Voyage. If a Ship is to be from Land two Months, and the Watch errs four Seconds of Time per Day different from what is supposed, the Error in that time amounts to a Degree of Longitude. If the Ship is longer time from Land, under such Error of the Watch, the Error of Longitude will be greater. If the Watch has a greater Error daily (and such may be) the Consequences of depending on the Watch will be accordingly.

Third Method; by the Moon.

1. The original Causes of Accuracy and likewise of Error by this Method, have been before considered in this Work; the Observations of Navigators who have determined the Longitudes of Places by it both on Land and at Sea, confirm its Utility in long and dangerous Voyages.

2. When this Method is practised at Sea by Persons who are not provided with proper Instruments to make the Observations, nor proper Books for making the Calculations, they cannot expect Success; these the Observer should have, and know the Use of both. Some Navigators come within forty Miles, others within twenty Miles, others within ten, and others within five Miles, by this Method; the Method itself, under its most favourable Circumstances, seems not to admit of Error greater than Twenty Miles, this is verified by able Observers.

Fourth Method; by the Variation.

1. This Method has greater Extremes than the former. At some Places the Ship may keep the same Latitude several Degrees without Alteration in the Variation; at such Places the Longitude cannot be taken by this Method without erring some Degrees, however truely the Variation Chart is drawn. At other Places, where the Variation Lines are most favourable, the Longitude may be had from them within a Degree or Sixty Miles of Distance.

CXXXVIII.

Directions for adjusting either the Octant or Sextant, in Order to make an Observation with it, for the Latitude or Longitude at Sea.

1. There have been such Alterations made by Instrument Makers, in the Parts of this Instrument, since its first Invention, and are still making, that an Account of them all cannot be expressed in a few words. Notwithstanding those Innovations, and the Instrument's being varied in its Construction, it depends on its original Principles, and the Adjustment of it in all its Forms, will ever be the same.

2. The Foot of the Index Glass's Cell, should be formed at Right Angles to the Plane of the Cell that is to hold the Glass; and should screw down at the Centre of the Index. The Glass should go in and out of the Cell freely; and without being liable to be out of the Perpendicular when fixed in the Cell.

3. The Eye Hole should be the same Distance from the Plane of the Instrument as the Line between the quicksilvered and unquicksilvered Part of the small Mirrour.

4. To adjust the Instrument. Put the Beginning of the Index to the Beginning of the Arch; then hold the Instrument upright, and move the small Mirrour vertically, until an horizontal Line (at a Distance not less than a Mile) appears one continued Line, across the quicksilvered and unquicksilvered Parts of the small Mirrour. For the horizontal Adjustment; hold the Instrument horizontally, and move the small Mirrour horizontally, until that horizontal Line, appears as one continued Line both on and off the quicksilvered part of the Small Mirrour.

5. In this State of the Instrument, the Adjustment is the same by the Sun (with a dark Glass next the Eye) the Moon, a Star, or any other remote Object; any Point in the Object, appearing both by direct Vision and double Reflection, as a Point in the Line between those two Parts of the small Mirrour. This Adjustment is of the greatest Consequence previous to an Observation; and if the Instrument is constructed after any other than the usual Manner, it must answer these Properties.

6. The Precepts for taking the Latitude by Meridian Altitudes taken with this Instrument are in Section 53. For Solar Time by the Sun, 54. Solar Time by a Fixed Star, 55. Amplitude, 56. Azimuth, 57.

CXXXIX.

Directions for observing with the Azimuth Compass, in order to take the Variation at Sea.

1. The Azimuth Compass has had such Alterations made in its Parts of late Years, that to give a particular Account of them, would require many words. The whole Use of this Instrument, in any of its forms, is to take the horizontal Bearing of the Sun or any other Celestial Body, with the north and south Line of the Magnetic Needle, at the Time when an Altitude is taken of the same Celestial Body; in order to determine the Variation of the Needle (or Compass Card) from the true Meridian, at the Place of Observation.

2. In the Use of this Instrument, when an Observation is to be made with it, the Needle or Card in the Box is suspended freely on its Pivot, and the Frame of the Instrument with its Divisions is to be moved, so that the Beginning of the Circumference on the Frame may agree as exactly as possible with the north or south Point of the Needle or Card; at the same Time, an Index (being the Base of an upright Plane) is turned toward the Sun's Centre, and the Degrees on the horizontal Frame, intercepted between the Index and the Point of the Card, are those to be taken.

3. Having the Altitude and this bearing the Variation is found thus. By the Co-latitude, Co-altitude, and Polar-distance, the true Azimuth is to be found; this being compared with the Bearing (commonly called the Magnetic Azimuth) gives the Variation.

4. In taking the Bearing by this Instrument, if the Needle or Card vibrates, the Point of the Frame should be kept as exactly as possible in the Middle of the Vibrations; and when several Bearings and Altitudes have been taken, at the Ends of short Intervals of Time, a Medium of them will give the Variation near the Truth.

5. After this manner, the Bearing is taken from the East or West Points of the Compass Card, when the Variation is to be taken at Sun-rising or Sun-setting, by an amplitude; then, the true and magnetic Amplitudes being compared, give the Variation. Here, the Sun's Centre is in the Horizon of the Sea, when his lower Limb appears Half the Diameter above that Horizon, supposing there is no Dip; if there is Dip, it must be cleared.

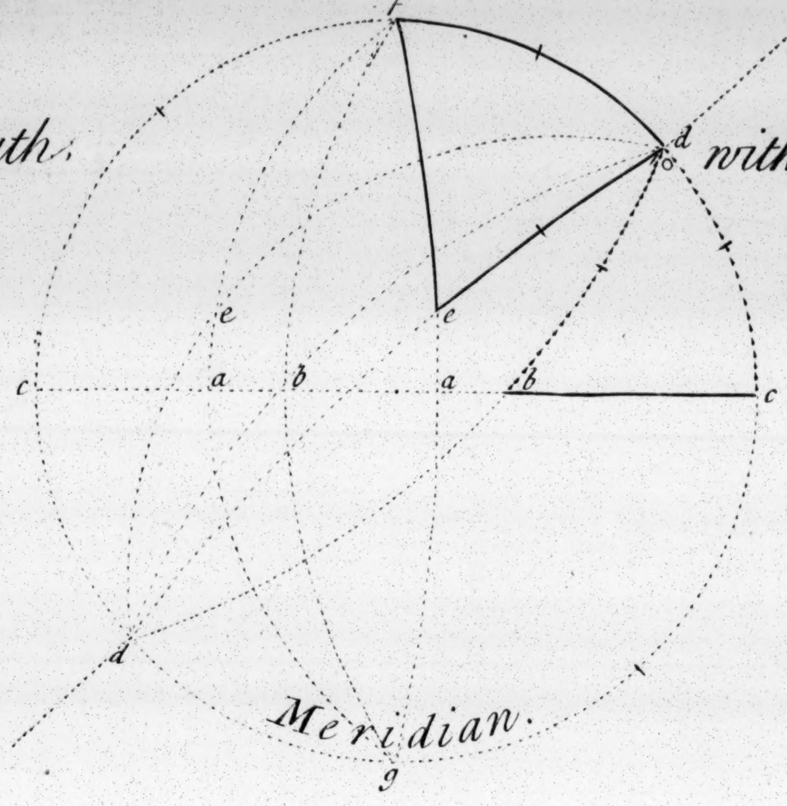
CXL.

Having the Latitude of a Place, and the Time Elapsed from the Instant the Sun is in the true Horizon; to determine the true Azimuth from the Point under the elevated Pole.

1. This Problem has two Cases; *1st.* when the Latitude and Declination are alike, *2d.* when they are unlike; the Circumstances attending these Cases are to be observed. In this Problem, two Sides and the included Angle of a spherical Triangle are given, and when the Sum of the containing Sides is more than a Semicircle, the supplemental Triangle is to be computed, and the Answer

The Azimuth without Altitude.

Given,
dc Latitude.
db á Pole,
de á Pole,
∠ bde.



Required,
the Azimuth
∠ dfe or ca
Also ae the
Altitude.

Calculation

dc Tang.	50°. 0′	10.07619	
db Co tan.	77 . 42	9.33853	
∠ bde Cos	74 . 56	9.41472	
∠ bde add	15 . 0		
Sub	89 . 56		
	180 . 0		
∠ edf	2)90 . 4		
its half	45 . 2		

de	77°. 42′	
df	40 . 0	
	2)117 . 42	
	58 . 51	half Sum Sides.
	2)37 . 42	
	18 . 51	half diff. Sides.

Sine	58°. 51	9 . 9324
Sine	18 . 51	9 . 5093
Co tan	45 . 2	9 . 9995
Tang	20 . 40	9 . 5764

Co sine	58°. 51′	9 7137
Co sine	18 . 51	9 9761
Co tan	45 . 2	9 9995
Tang	61 . 19	10 2619
	20 . 40	
	81 . 59	∠ efd required

When the Declination & Latitude are unlike be will fall below the Equator, & when the Sum of de & fe exceeds 180, the supplemental Triangle deg must have its angle g found, this is supplement to dfe.

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Anſwer will be accordingly. See Figure, Azimuth without Altitude.

Caſe 1.

2. In the Triangle *bdc*, *dc* is 50° 0′; *db* is 77° 42′; and the Angle at *c* is a Right Angle. Therefore, the Tangent of 50° 0′ added to the Co-tangent of 77° 42′ makes the Coſine of the Angle *bdc* added to Index 10, as taken from the Logarithms. Hence, the Angle *bdc* is 74° 56′ which being added to the Angle *bde* (15° 0′) makes the Angle *cde* 89° 56′, and this taken from 180° gives the Angle *edf* 90° 4′. Then, having the Co-latitude *df*, the Polar-diſtance *de*, and the included Angle *edf*, one of the other Angles *dfe* is found 81° 59′; or *ac* the Azimuth required. The Proportions for the Sides and included Angle, are in Section 41. The Half Sum of the Sides is 58° 51′; their Half Difference is 18° 51′. What makes this Problem preferable to ſome others deſigned for getting the Variation by Obſervations when the Sun is very near the Horizon, is its Accuracy; for, having once determined the Inſtant of the Sun's being in the Horizon, the Truth comes out independent of Refraction and the horizontal Vapours.

Caſe 2.

This is when the Latitude and Declination are one of them North, the other South. Here, the ſupplemental Triangles to a Semicircle are to be ſolved, although they fall below the Horizon, toward the depreſſed Pole.

After the ſame manner any other Celeſtial Body may be obſerved and applied, having firſt reduced the meaſured Elapſed Time, to the Equatorial Motion.

CXLI.

Of Problems with auxiliary Data, in the four uſual Methods of determining the Latitude and Longitude, of a Ship at Sea.

1. When (in any of the uſual Methods of finding the Longitude at Sea) the previous Obſervations have been made, and they are deficient in their Number, or are any otherwiſe very defective in ſome particular Part; either the Longitude cannot be deduced, or the Deficiency muſt be ſupplied by ſome other things that are known; theſe may be called auxiliary Data.

2. Four Problems of this kind have been ſuppoſed to happen in the Lunar Method, namely, by a Deficiency of the Sun's Altitude when the Moon's Altitude and the Diſtance have been taken, a Deficiency of the Star's Altitude when the Moon's Altitude and the Diſtance have been taken, and a Deficiency of the Moon's Altitude when either the Sun or Star's Altitude and the Diſtance have been taken. Here, other things ſuppoſedly known, namely, the Solar Time at the Ship, the Ship's Latitude and Longitude, at the time of Obſervation, have been uſed as auxiliary Data, for finding the Altitude of the Sun, Star, and Moon, in order to be applied with the two other cotemporary Obſervations. But, 1*ſt*.

3. At Sea, when the Altitude of one of the Luminaries can be taken, the Altitude of the other can be taken, with no great Difference in Accuracy. 2*d*. An Error in the Sun or Star's Altitude, although it be great, may not produce a great Error (eſpecially when it is taken nearly eaſt or weſt) becauſe, it chiefly affects the Ship's Time only. 3*d*. In the auxiliary Data, the ſuppoſed Time at the Ship, muſt have been deduced from other things, ſuppoſed true, whilſt the ſuppoſed Latitude and Longitude may be both erroneous; and therefore, not one of the three auxiliary Data, are to be depended on. If the ſuppoſed Time at the Ship, the Ship's Latitude or its Longitude, are either of them erroneous, the deduced Altitude is erroneous. To this Error there is no apparent Limit, whilſt the Error by an obſerved Altitude cannot (in uſual Caſes) be greater than what ariſes from a Part of the apparent diurnal Arch applied to a falſe Horizon. Farther.

4. In uſual Caſes, at Sea, the Moon will want the leaſt Aſſiſtance from ſuch Data; in the Night, that part of the Horizon which is beneath her, will frequently be better illuminated than the other Parts, and if her Altitude is got this way, there muſt be a previous Correction for Parallax, and other things which moſt Perſons are unwilling to meddle with. The Sun's Altitude, never wants this Help as much as that of the Moon; the Zodiacal Stars when nearly under or above the Moon, never want it more than the Moon; and under other Poſitions, the Horizon is often illuminated ſtrongly enough for a Dependance on the Altitude, as taken with the Inſtrument.

5. If the Altitude is wanted from the auxiliary Data, it may be eaſily had by application of ſpherical Triangles or the Precepts concerning them in Section 41 of this Work. If the Solution is purſued by ſuch a Method, there can be no Ambiguity or Uncertainty concerning the Anſwer; for, the Co-latitude and two Polar-diſtances of the Celeſtial Bodies, alſo, their Equatorial Diſtances, and what each of them is either paſt the Meridian or ſhort of it, are eaſily to be derived from the Data, and by a Calculation no way difficult, the Co-altitude may be had which was required. In ſuch a Method of Solution, the Tables of Log. Sines and Secants are all that are wanted, and to but four Places of Figures beſides Index, in uſual Caſes, but if greater Accuracy is wanted, five or ſix Figures may be applied, and ſuch other Corrections as are neceſſary for getting the Altitude as it would have been obſerved on the ſpheroidical Earth or Sea.

EXAMPLE I.

True Latitude N.	49° 57′	
Sun's Decl. N.	19 34	
Sun past Noon	75 55	

Lat. N.	49° 57′	Cosine	9.80852
From Noon	75 55	Sine	9.89675
First Arc	38 37	Sine	9.70527
Lat. N.	49 57	Sine	9.88394
First Arc	38 37	Sec. l. R.	0.10716
Second Arc	11 34	Cosine	9.99110
Polar-dist.	70 26		
Remainder	58 52	Cosine	9.71352
First Arc	38 37	Cosine	9.89284
Sun's Alt.	23 50	Sine	9.60636

EXAMPLE II.

True Latitude N.	51° 32′	
Sun's Decl. S.	20 38	
Sun short of Noon	50 22½	

Lat. N.	51° 32′	Cosine	9.79374
From Noon	50 22½	Sine	9.88662
First Arc	28 38	Sine	9.68036
Lat. N.	51 32	Sine	9.89374
First Arc	28 38	Sec. l. R.	0.05662
Second Arc	26 52½	Cosine	9.95036
Polar-dist.	110 38		
Remainder	83 45½	Cosine	0.03632
First Arc	28 38	Cosine	9.94338
Sun's Alt.	5 29	Sine	8.97970

EXAMPLE III.

True Latitude N.	51° 32′	
Aldeb. Decl. N.	16 3	
Aldeb. past Meridian	43 56	

Lat. N.	51° 32′	Cosine	9.79383
From Merid.	43 56	Sine	9.84125
First Arc	25 34	Sine	9.63508
Lat. N.	51 32	Sine	9.89374
First Arc	25 34	Sec. l. R.	0.04475
Second Arc	29 47	Cosine	9.93849
Polar-dist.	73 57		
Remainder	44 10	Cosine	9.85571
First Arc	25 34	Cosine	9.95525
Aldeb. Alt.	40 19	Sine	9.81096

EXAMPLE IV.

True Latitude S.	14° 45′	
Moon's Decl. N.	10 11	
Moon past Meridian	46 28	

Lat. S.	14° 45′	Cosine	9.98545
From Merid.	46 28	Sine	9.86032
First Arc	44 31	Sine	9.84577
Lat. S	14 45	Sine	9.40586
First Arc	44 31	Sec. l. R.	0.14688
Second Arc	69 4½	Cosine	9.55274
Polar-dist.	100 11		
Remainder	31 6½	Cosine	9.93257
First Arc	44 31	Cosine	9.85312
Moon's Alt.	37 37½	Sine	9.78569

6. In words at Length, the Rule by which these Operations are made, is thus.

1st. Get the true Latitude of the place of Observation, the true Declination of the Celestial Body whether it be Sun, Moon or Star, and the Time in Degrees and Minutes that it is either past the Meridian or short of it. *2d.* Add together, the Cosine of the true Latitude, and the Sine of the distance from Noon, the Sum (rejecting Radius) is the Sine of Arc the First. *3d.* Add together, the Sine of the true Latitude, and the Secant less Radius of Arc the First, the Sum (rejecting Radius) is the Cosine of Arc the Second. *4th.* Take the Difference between Arc the Second and the Polar-distance of the Celestial Body; then, add together the Cosine of this Difference (or Remainder) and the Cosine of Arc the First, the Sum (rejecting Radius) is the Sine of the Altitude required.

7. However easily such Data as the foregoing may be applied, the Observer will do well to take the cotemporary Observations as accurately as he can, and depend on them for Success. The hidden Errors that are in such Data, may be greater than he supposes, and it may require no little thought to consider the Origin of them and to judge of their Amount.

8. As the Lunar Method have such Data, so have the Methods, by Course and Distance sailed, by the Watch or Time-keeper, and by the Variation; even that by Jupiter's Satellites on Land, may depend on a supposed regular Gain or Loss of Time by the Clock, from the Instant it was supposed right, whilst neither of them may happen to be the Truth, so the Longitude inferred may be proportionably erroneous. In the Watch Method of finding the Longitude, when the Machine has been carefully tried for a long time before the Voyage, and is carefully kept at Sea, there may then be hope of its continuing to go on as before; but, if it happens to the contrary, after the utmost Skill and Care that Man is master of, have been exerted in its Formation and Preservation, its Defects plainly shew that it can no way compare with the Motions of the Celestial Bodies, in keeping the Divisions and Subdivisions of Time.

CXLII.

Directions for finding whether a Watch or other Time-keeper keeps Equal Time, and what its Error is; by observed Transits of the Sun and Fixed Stars.

1. It being certain, that a Watch or Time-keeper, when taken to Sea may go either right or wrong, the Person who is to apply it should know how to find its Defects on Land, before it is

is at Sea and deceives him. In my Practical Aſtronomy, a Method was ſhewn how to do this to great Exactneſs without the Uſe of Inſtruments. Thus,

2. Over an horizontal Line that is nearly in a north and ſouth Direction, or in other words, near the Plane of the Meridian, let two perpendicular Threads be tightly ſtrained at the Diſtance of, from one to two, three or more Feet from each other, whereby to take the Meridian Tranſits of the Celeſtial Bodies, and compare them with the Times ſhewn by the Watch, Time-keeper or Clock. See Equation of Time Tables.

3. The Table of the Equation of Time, ſhews plainly whether the Sun's meridian Tranſit is before or after the Equal Time to be ſhewn by the Time-keeper and how much, whether it be additive or ſubductive; from which, a Method of trying it by the Sun's Tranſit will be as follows.

4. Firſt let the Minute and Second Hands of the Time-keeper be put as many Minutes and Seconds before or after Twelve as is ſhewn by the Equation Table. Secondly, the ſame Day, let the Obſerver withdraw himſelf one, two, or more Feet, behind the ſtrained perpendicular Lines, and note the Inſtant when the Sun is biſected by them both; and then, let the Time-keeper go on freely.

5. One, two, or more Days after, let the Sun's Meridian Tranſit be taken in the ſame manner, and the Time noted as ſhewn by the Time-keeper. Then, if it has gone to Equal Time in the Interval, it will differ from the Time of Obſervation that Day, by the Equation of Time for the ſame Day; otherwiſe, it has gained or loſed in its rate of going; and the daily Gain or Loſs may be thereby judged of and noted, to be allowed for in future.

6. By the ſame Method, the Fixed Stars may be obſerved and compared, but their Right Aſcenſions muſt be carefully compared with the Right Aſcenſion of the Sun, otherwiſe the Accuracy which this Method admits of will be loſt, for without ſuch Allowance they will only ſhew the Gain or Loſs on Siderial Time.

EXAMPLE I.

March 24, Sun South	12h	8′	30″
Should be South at	12	6	21
Clock before Equal Time		2	9

EXAMPLE II.

March 24, Procyon at	7^h	29′	27″
Should be on Merid. at	7	27	20
Clock before Equal Time	0	2	7
March 24, Regulus at	9	58	19
Should be on Merid. at	9	56	13
Clock before Equal Time	0	2	6

By ſuch Obſervations, it will appear how accurately the Time-keeper may be expected to keep to mean Solar Time at Sea, and what Allowance ought to be made for its gaining or loſing, during any Time whilſt the Ship is at Sea.

CXLIII.

Inſtructions for finding the Longitude; by the uſual Method of Sailing.

1. The principal Errors which ariſe in this Method are in the Courſe ſailed, the Diſtance meaſured, the Variation, Leeway and Currents not properly allowed for; therefore the Navigator ſhould, with all poſſible Care and Attention, endeavour to prevent all Errors arriſing from theſe Cauſes. For that purpoſe

2. Every Hour in the Day and Night, the Courſe ſailed by Compaſs, the Diſtance run, and the Leeway, ſhould be taken if they can be taken, and with as much accuracy as poſſible, In the Night and at other times when they cannot be taken, the beſt Allowances that can be judged of ſhould be made for them. Theſe, with the allowed Variation, and ſuch other Particulars as may affect either the deduced Longitude or Latitude, ſhould be inſerted in the Journal in order to infer the Latitude and Longitude come to, by the uſual Method of Days Works; and every time that the Ship alters her Courſe, it ſhould be inſerted as for the Hour neareſt to that Change.

3. All Opportunities for taking the Latitude by Meridian Altitudes, of the Sun, Moon, Saturn, Jupiter, Mars, Venus, and the Fixed Stars of the firſt Magnitude, ſhould be carefully attended to; and the Latitudes by them inſerted in the Journal.

4. Whilſt the Latitude appears well known from Meridian Altitudes, there is the leſs occaſion for ſeeking it by any other Methods; but, when ever the Journal is defective in this, two Altitudes and the Elapſed Time, two Stars one near the Meridian, or any other Method, ſhould be applied for recovering the Latitude as ſoon and as accurately as poſſible.

5. When it is either known or ſuſpected that the Ship is acted on by a Current, the beſt Methods ſhould be taken for knowing its Courſe and Velocity, eſpecially when it runs ſo as to admit of no apparent Effect upon the Latitude by Reckoning, or makes it agree with that by Obſervation.

6. The Latitude by Account and that by Obſervation ſhould be kept apart from each other, and when they differ conſiderably, a Correction of the Longitude ſhould be made, by help of the obſerved Latitude and other Data moſt to be depended on. The Latitude and Longitude thus inferred, ſhould be taken for the Truth, till there

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is another Difference between the Latitude taken and the former observed Latitude thus carried on by Account; and so on to the end of the Voyage.

7. In deducing the Longitude from Day to Day, at every particular Course the most proper Allowances should be made for Variation, Leeway, Currents, and the true Course found to the nearest Degree; with which and the Distance, the Differences of Latitude and Departure may be taken out by Inspection, and the true Difference of Longitude made, had by either Middle Latitude or Mercator's Sailing. The Difficulties which attend the clearing of each Course accurately, and the Errors that must arise for want of it are obvious; these are easily to be made less if not entirely removed, by sailing according to the following Instructions.

CXLIV.

Instructions for finding the Longitude; by Magnetic Sailing.

1. In the usual Method of Sailing, the Courses are taken hourly by the Compass Card, the Distances are measured, and Allowances are made for Variation, Leeway and other Deceptions on each Course and Distance; after this, the whole true Difference of Latitude and Departure made good are had, and thereby the Difference of Longitude. If this is done many Times in the whole Day, the Computation will be much and the Longitude but imperfectly determined.

2. In Magnetic Sailing the Courses are taken hourly by the Compass Card, the Distances are measured, and Allowances are made for Leeway and other Deceptions on each Course, except the Variation; after this, the whole Magnetic Difference of Latitude and Departure made good are had, and thereby the Magnetic Course made good and true Distance made good, for the end of every twenty four hours. In the Interval, the true Mean Variation is to be got as correctly as possible, and to be added to or subtracted from the Magnetic Course made good (as the Case requires) to get the true Course made good, with which and the true Distance made good, the true Difference of Latitude and Departure are to be found, and thereby the true Difference of Longitude.

3. In this Method of Sailing and making up the Account, the Variation is supposed to be taken by a Compass agreeing with that by which the Ship is steered; and therefore, however erroneously it is constructed, no Error concerning Allowances for Variation can arise therefrom, nor any Difference on this Account, when several Ships sail either together or apart, by Compasses that are each of them erroneously constructed.

4. This Method likewise takes off the trouble and Error in working single Courses; the Parts of the Process which are not to be carried on into the Longitude, may be taken out by Inspection from the Table of Difference of Latitude and Departure, and those that are carried on into the Longitude, are accurately and easily to be worked by three or four Places of logarithmic Figures, without the Meridional Parts, as in the Instructions for those Purposes. See those Instructions.

5. Here, three, four, five, or six Figure of Logarithms may be applied the latter of which will be One hundred times more exact than four Figures of Meridional Parts, in Courses very nearly east and west.

CXLV.

Instructions for finding the Longitude; by the help of Watches or Time-keepers.

1. The principal Instruments and Books which the Navigator should be provided with in the Practice of this Method are, the Sextant or Octant accurately made and adjusted, the Watch or Time-keeper designed for keeping mean Solar Time, the Ephemeris for taking out the Sun's Declination to the nearest Minute of a Degree, also the Right Ascensions and Declinations of the principal Fixed Stars to the nearest Minute of a Degree for the present Time, and Logarithmic Tables for making the Computation.

2. Before the Time-keeper is taken to Sea, the Navigator should know that it has kept nearly to mean Solar Time during a considerable Interval, otherwise he may not reasonably expect it will keep such Time during a much longer Interval or a long Voyage. In the Trial of such a Machine, he is not to conclude that Observations made inaccurately will be sufficient, nor any thing more imperfect than the best Methods for measuring Intervals of Time. If it has been compared in its going with a Regulator having the gridiron Pendulum, and agreed therewith or very nearly in its rate of going or losing, for a Month or longer, it may be supposed proper for the Longitude, and so it may if it has been found to keep nearly the same Rate of gaining or losing, by Meridian Transits of the Sun or Fixed Stars; this being determined, whatever it gains or loses per Day on mean Solar Time, should be known to the Navigator, and allowed for every Day throughout the Voyage.

3. In this Method all Opportunities should be taken for having the Latitude correctly known from Astronomical Observations, as often as possible by Meridian Altitudes of Sun, Moon, Planets and Fixed Stars; and when such cannot be had, any of the Methods should be applied which have been

1st A Calculation at Large.

Given, the Ship's Lat.d by Observation 34°.17′ N. Central Dist. 74°.16′ at Gr. 6h.

Dist.ce ☉ & ☽'s nearest Limbs Obs.d 73.44.27″. Semid.rs 31′.36″ } by Watch

Alt.d of ☉'s l.r L.b above the Sea Obs.d 22.3, height of the Eye 18 Feet. } 4h.50′.57″

Alt.d of ☽'s l.r L.b above the Sea Obs.d 80.41, height of the Eye 18 Feet. }

Alt.d of ☉'s l.r L.b above the Sea Obs.d 19.13, a Correct Observ.n at 5h.4′.38″

April 4th 17 Afternoon. The other Data are as under. Req.d the Long.d?

Thus a Watch shews Time at the Ship in the Lunar Method but it is best had by the true Altitude when the Distance is taken. That Time in Degrees compared with Greenwich Time in Degrees, gives the Longitude easier & truer without depending on the Watch or the Ship's Run between the Observations.

2.dly For the Refraction.

22°.15′ }
80.53 } = 2.2293

74.16.3″ = 10.0165

74°.16′ } A = 176″ = 2.2458

22.15 } B = 32″

144″ = 2′.24″

See Formula 74°.16.3

D = 74°.18′.27″

& For the Parallax.

56′.12″ = .5055

22°.13. = 10.4223

74.18.27 = 9.9835

1.st Arc = 22′.5″ = 0.9113

56′.12″ = .5055

80.53. = 10.0055

74.18.27 = 10.5512

2.d Arc = 15′.36″ = 1.0622

C = 6′.29″

D = 74°.18.27

E = 74°.11′.58″

74°.18′ }
10′ } = 0″ } = F = 0″
6′ = 0″ }

P = 74°.11′.58″

See Formula

3.dly For the Time at Greenwich.

3.h = 73°.1′.27″

P = 74.11.58

1°.10′.31″ = .4070

3.h = 73°.1′.27″

6.h = 74.28.50

1°.27′.23″ = .3138

2h.25′.14″ = .0932

3h

See Ephem.s

G = 5.h25′.14″ = 81°.19′.

4thly For the Time at the Ship.

See Diagram.

Lat.d = 34°.17

PZ = 55.43

☉'s Decl.n = 5°.48′

PS = 84.12

☉'s Alt.d = 19°.22′

ZS = 70.38

Spherical Triangles.

PZ = 55°.43 Co. Ar. .082882

PS = 84.12 Co. Ar. .002229

ZS = 70.38 9.984397

2) 210.33 Sine 9.754595

74°.44 = 105.16 Sine 19.824103

34.38

35°.15′ = Cosine = 9.912051

2

70°.30′ = 4h.42′.00″ by Observation

5.4.38 by the Watch

Watch 22′.38″ too Fast.

4h.50.57

S = 4h.28′.19″ = 67°.5′.

14.14

5thly For the Longitude

Time of 3 Obs.s at Greenwich 5h.25′.14″

Time of 3 Obs.s at the Ship 4.28.19

Longitude L = 14°.14′ W = 0h.56′.55″

Ship West of Greenwich.

Improved & Published as the Act directs June 14. 1786, by Samuel Dunn.

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been delivered in this Work. The mean Solar Time kept by the Timekeeper is to be for the Ephemeris's Meridian, and the Sun's Declination to be taken out for the Day and Hour ſhewn by the Time-keeper, when the Latitude is to be inferred from Meridian Altitudes of the Sun.

4. At the Time of obſerving for the Longitude, the Hour, Minute and Second, ſhewn by the Time-keeper are to be accurately noted, at the ſame Inſtant the Altitude of the Sun's lower Limb is to be taken. Then, by the Altitude of the Sun cleared from Semidiameter, Dip and Refraction, the Declination taken out for the Hour ſhewn by the Time-keeper, and the true Latitude of the Place, there are the Co latitude, Polar-diſtance and Co-altitude, from which (by Section 54) the Time at the Ship is found, and this added to or ſubtracted from the Time noted by the Time-keeper, is the Longitude from the Ephemeris's Meridian.

5. Such a Time-keeper may be otherwiſe uſeful in a Ship at Sea, for determining the Latitude, having Meridian Altitudes of the Sun near the Equinoxes, the Moon, and the Primary Planets; two ſuch Altitudes happen every Day by the Sun and Moon, and four may happen by the larger Primary Planets, and their Declinations are all to be taken out for the Hour at the Ephemeris's Meridian. Therefore, a common Watch kept nearly to ſuch Time may be uſeful in a Ship at Sea; but, to ſhow nearly the Time of Meridian Tranſit, it muſt not be far different from the Time at the Ship.

CXLVI.

Inſtructions for finding the Longitude at Sea, having the Latitude, Sights of Sun and Moon, and an Ephemeris.

1. In order to take the Longitude as correctly as poſſible by this Method, the Perſon who is to take the Diſtance of Sun and Moon's neareſt Limbs, ſhould be provided with a proper Sextant or Octant himſelf, he ſhould likewiſe be aſſiſted by two Perſons provided with good Octants to take the Altitudes, and ſuch other Perſons ſhould aſſiſt as may be neceſſary for preventing Errors and making the Obſervations as expeditiouſly and correctly as poſſible.

2. The Inſtrument to be uſed in taking the angular Diſtance of the Limbs, ſhould not be leſs than Fifteen Inches Radius; becauſe, if it be leſs, the Diviſions by the Nonius on the Limb, will be ſo ſmall, that an Obſerver or any other Perſon who reads for him, may be apt to miſtake a Minute of a Degree in reading (eſpecially if the chamfered Edge of the Index doth not come cloſe to the Plane of the Limb) and he may reaſonably expect to come to that Accuracy in many Obſervations with the naked Eye, when the Opportunity is the moſt favourable.

3. He ſhould be careful to have an Inſtrument that is well centered, formed in that Part by the moſt perfect Turnery out of ſolid Metal, ſo that the Index be not liable to bend nor ſhift its Centre. The other Parts of the Inſtrument ſhould be well ſecured from being effected by Heat or Cold, Drought or Moiſture. The Glaſſes, particularly the dark Glaſſes, accurately ground; the Diviſions correct; the Teleſcope (if any to the Inſtrument) not liable to ſtart from its right Poſition, and ſuch other good Qualities belonging to the Inſtrument, that it may always anſwer under the ſevereſt Examination, to the neareſt Minute of a Degree.

4. With ſuch an Inſtrument, and without a Teleſcope near the Eye, the angular Diſtance of Sun and Moon's neareſt Limbs may be taken ſufficiently exact for the Longitude, provided the Obſerver takes ſeveral Obſervations quickly and with much care, and all within a ſhort Interval of Time. Here will be no more than three Glaſſes, namely, the Index Glaſs, the ſmall Glaſs, and the dark Glaſs.

If the Teleſcope be applied inſtead of the Plain Sight, the Contact of the Limbs will appear plainer, and the Diſtance will be more accurately taken, provided there is no additional Error by its Glaſſes or their Poſition to the Plane of the Inſtrument.

5. As the Perſon who takes the Diſtance is the principal Obſerver the other two are to obſerve his Signals or Words, for he is to take a Diſtance, ſpeak at that Inſtant, write or cauſe to be written down the Degrees and Minutes, proceed to take another, and another, as correctly and expeditiouſly as poſſible at the end of nearly ſmall and equal Intervals of Time.

6. Whilſt the angular Diſtances are taking, the two Aſſiſtants muſt (with all attention) wait for the Inſtant when a Signal is announced or ſpoke by the principal Obſerver. When he ſpeaks, each of them muſt be ready, the one with the Sun's lower Limb, the other with the Moon's lower or upper Limb at the Sea, and they are to write down their obſerved Altitudes each one apart.

7. The Sum of all theſe Diſtances of Limbs divided by the Number of them gives the obſerved Diſtance of Limbs or the firſt of the three cotemporary Obſervations. The Sum of the Altitudes of the Sun's lower Limb divided by the Number of them, and this cleared from Semidiameter and Dip, gives the ſecond of the cotemporary Obſerva-

tions. The Sum of the Moon's Altitudes divided by the Number of them and this cleared from Semi-diameter and Dip, gives the third of the cotemporary Obfervations, thus when the Linear Tables are to be applied; but, when the Method by Sun and Moon's Angles, the two Altitudes muft likewife be cleared from Refraction.

8. As the Affiftants may be fuppofed to be provided with Watches, and fuch as are ufually worn, they may (if they pleafe) make the following Ufe of them; not that thefe cotemporary Obfervations require any fuch additional Affiftance, but that they may thereby be taken at more equal Intervals, and likewife at more proper times of the Day. Their Ufe is thus. Let one of thefe be fet fomewhat near to the Ship's Time, the other fomewhat near the Ephemeris's Time, both to the fame Minute tho' not the fame Hour, this may be readily done, the former by the Ship's Clock, the latter by the rough central Diftance taken a little before the nicer Diftances are taken, their Ufe will be thus.

9. By the Watch for the Ship's Time, it will be known what Equatorial Diftance the Sun and Moon are from the Meridian in a rough manner, and thereby when are the moft proper times for obferving, if the Calculations are to be made by any of the Methods before fhewn in this Work; likewife the moft proper Pofitions of the Sun for two Altitudes and the Elapfed Time, and for Meridian Altitudes when they can be obferved. The Watch for the Ephemeris's Time, will readily fhew the Hour for which the Declinations and Right Afcenfions are to be taken out, this will be ufeful in taking the Latitude in the Night by two Stars.

10. Having the three cotemporary Obfervations, by them, the Linear Tables (or otherwife if the Method by Sun and Moon's Angles) the Tables of Common Logarithms, Log. Sines, Tangents and Secants, together with the ufe of the Ephemeris, the true Diftance of Centres is to be found as in the Example of Section 104; then, the Time at the Ephemeris's Place as in Section 108; then, the Time at the Ship's Place as in Section 111. The former Method of each Example, may be applied when thegreateft Accuracy is not required, but when it is required the Operation fhould be as for the Sphere. In this Cafe, the Obferver will be often able to avoid any Errors that may arife from the Spheroidical Figure of the Earth, by making his Obfervation as near the Prime Vertical or eaft and weft Azimuth Circle as poffible, the Confequences of which have been already fhewn. Then, as in Section 112, the Sum or Difference is the Longitude.

11. Here, it feems proper to take notice of a Practice which has been continued and is thought effentially neceffary by many Perfons, in taking the Longitude at Sea by feeing Sun and Moon in the Day time. Such Perfons get the three cotemporary Obfervations and thereby the Time at the Ephemeris's Place, this Time they compare with a Watch that has been fet to the Ship's Time, and the Sum or Difference they will have to be the Longitude; but, this has been confidered in Section 103.

12. The Method which all Navigators practice in keeping their Journals, is to afcertain the Ship's Latitude as accurately as they can for the Noon of each preceding Day, either by actual Obfervations or by the Ship's Reckoning; and therefore by infpecting the Latitude at a former Noon and the intermediate Runs, the prefent Latitude may be nearly known, this is the Latitude to be ufed together with the cotemporary Obfervations in order to take the Ship's Longitude without ufing the Watch in order to fhew the correct Time at the Ship, and that there can be no better Datum for the Latitude in order to fet the Watch to the Ship's Time, is evident from the Problem itfelf.

13. A Calculation, after the three cotemporary Obfervations are taken, the Latitude is known, and the rough hour with its Dependencies are taken out, is thus.

Obf. Dift. of Centres			74° 16′ 3″
Sun's Altitude cleared			22 15
Moon's Altitude cleared			80 53
N°. in Table I.			2. 229
Co-ar. of Dift. Centres			0. 017
Common Log. of	176″	Sum	2. 246
N°. in Table II.	32		
For Refraction	144	makes	2 24
Obf. Dift. of Centres			74 16 3
Dift. cleared from Ref. D.			74 18 27

N°. D.	74 18 27	Sine	9.9835
Sun's Alt.	22 13	Co-fec.	10.4223
Hor. Par.	56 12	Pr. Log.	0.5055
Firft Arc	22 5	Pr. Log.	0.9113
N°. D.	74 18 27	Tang.	10.5512
Moon's Alt.	80 53	Co-fec.	10.0055
Hor. Par.	56 12	Pr. Log.	0.5055
Second Arc	15 36	Pr. Log.	10622
For Parallax	6 29		
Laft Correction	0		
True Dift.	74 11 58		

Firft Hours	3 Dift.	73° 1′ 27″
Second Hours	6 Dift.	74 28 50
Firft Hours	3 Dift.	73 1 27
True Diftance		74 11 58

Firft

Firſt Difference		1 27 23		
Second Difference		1 10 31		
Firſt Proper Log.				0.3138
Second Proper Log.				0.4070
	36 19	Pr. Log.		0.0932
Firſt Hours	45 0			
Ephem. Time	81 19	paſt Noon		
Co-lat. N.	55 43		Co-ar.	0.08288
Polar-diſt.	84 12		Co-ar.	0.00223
Co. Alt.	70 38			
Sum	210 33			
Half Sum	105 16		Sine	9.98435
Remainder	34 38		Sine	9.75459
			Sum	19.82405
	35 15		Coſine	9.91203
Ship's Time	70 30			
Longitude	10 49	Weſt of Ephemeris Place.		

14. By this Example and others that might be introduced, it is evident that before the Operation could be made with the Perſpecuity and Shortneſs that is in the foregoing, there was an abſolute neceſſity for putting, the Common Logarthmic Sines, Tangents, Secants, and Co-ars. into a better Form and more ready for Uſe; likewiſe, the Logiſtic Logarithms and all the other Tables into Forms very different from what are in other Books; even the principal parts of the Propoſitions from which the Precepts have been derived have been altered in their Inveſtigation and Application, in order to make it eaſier for the Computor. The Tables of Logarithms, the Linear Tables, the Treatiſe on Elapſed Time, the Formulæ, and other things publiſhed by me, are Proofs that they have been all abſolutely neceſſary, for clearing theſe Subjects from the many Difficulties and Incumbrances which they have been left under, by good aſtronomical Writers, both at Home and Abroad.

CXLVII.

Inſtructions for finding the Longitude at Sea, having the Latitude, Sights of the Moon and a Zodiacal Star, and an Ephemeris.

1. When the Longitude is to be taken by this Method, independent of a Watch to ſhew Time at the Ship, the parts of the Horizon beneath the Star and beneath the Moon, ſhould be well enough defined to take the Altitude of either of them without erring five Minutes of a Degree, this may not produce an Error of more than as many Miles, in an Oblique Sphere, when the Star is remote from the Meridian. If the Horizon is as well defined as it frequently is at Sea, the Error in Altitude may not be more than one Minute or Mile.

2. One principal Obſerver and two Aſſiſtant Obſervers at the leaſt, ſhould be employed in getting the cotemporary Obſervations, and a fourth may be of Uſe for aſſiſting them. The Sextants or Octants, ſhould have the good qualities mentioned in the former Section, and all the Obſervers ſhould act in Harmony with each other either by proper Signals or Words, at the Inſtants when the Diſtances and Altitudes are taken. If the Zodiacal Star appears large and the Horizon is well defined, both the Diſtance and the Star's Altitude may be taken by plain-ſighted Inſtruments, but otherwiſe the ſmall Teleſcope near the Eye may be wanted in the Uſe of both.

3. When the Moon's light is very ſtrong, the lighteſt dark Glaſs is to be turned down for leſſening it; then, the Moon's neareſt or fartheſt Limb is brought to the Star. If ſuch a Glaſs is behind the ſmall Mirror, the Star may frequently be brought to the Moon. After the ſame manner, the Moon may be brought to the Sun; but, in all Caſes, the more Glaſſes the Rays of Light are either reflected or refracted from, in the Inſtrument, the more Care muſt have been taken by the Inſtrument Maker, to have them all right, otherwiſe Error will follow.

4. Having the cotemporary Obſervations (taken after the manner directed in the former Section for the Sun and Moon) the Altitudes are to be cleared and the true Diſtance of Centres found; then, the Ephemeris's Time is to be found, next the Time at the Ship, and the Sum or Difference of theſe is the Longitude; as in Section 95.

CXLVIII.

Inſtructions for finding both the Latitude and Longitude at Sea; having Sights of the Sun and Moon, either when one of the Luminaries is near to or remote from the Meridian of the Place of Obſervation.

1. In either of theſe two Problems, the ſame Method is to be obſerved for getting the three cotemporary Obſervations, as has been particularized in Section 146; then, it is to be conſidered, what helps may lead the Obſerver to judge whether either of the Luminaries is near the Meridian or remote from it. If it is ſuppoſed near the Meridian, the Method in Section 125 is to be applied. If it is ſuppoſed remote from the Meridian, the Method in Section 129 is to be applied. In either Caſe, the true Diſtance of Centres, the Time at the Ephemeris's Place, the Latitude, and the Time at the Ship's Place are to be found, and the Sum or Difference of the two Times is the Longitude.

CXLIX.

Inſtructions for finding both the Latitude and Longitude at Sea; having Sights of the Moon and a Zodiacal Star, either when one of the Luminaries is near to or remote from the Meridian of the Place of Obſervation.

1. In either of theſe two Problems, the true Diſtance of Centres, the Time at the Epheme-

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ris's Place, the Latitude, and the Time at the Ship's Place are to be found; and thereby, the Longitude.

From what has been before delivered it is evident, that when the Method of finding the Longitude at Sea by the uſual Method of Sailing, is practiſed without any other Aid than what can ariſe from its own Principles, it will ever be defective in long Voyages, and that its Defects, will, at all times and places, receive great help and aſſiſtance from Aſtronomy. That when the Method of finding the Longitude at Sea by the help of Watches or Time-keepers, is practiſed without any other Aid than what ariſes from its own Principles, no Inferences whatever can be drawn, either concerning the gaining or loſing of the Time-keeper before it ſets out on a Voyage, during its being at Sea, or when it comes to the End of the Voyage, without the application of Aſtronomy. That the Lunar Method, of finding the Longitude at Sea, whether it be by the Sun and Moon or the Moon and Zodiacal Stars, has for its Foundation, the Laws of the Syſtem of the World, the Principles and Practice of Aſtronomy; and therefore, it may be reaſonably ſuppoſed, that the Principles on which this Method depends will always be invariably the ſame, and that the Practice of it can have no greater Difficulties than what ariſe in making the Obſervations and Calculations.

CL.

Inſtructions for finding the Longitude at Sea; by the help of Variation Charts.

1. In this Method, the Obſerver ſhould be provided with a good Sextant or Octant for taking the Altitude of Sun, Moon, Planet, or Fixed Star, a good Azimuth Compaſs for taking the Magnetic Bearing of the Celeſtial Body, proper Aſſiſtants, and Books for making the Calculation. In this Method, the beſt times are when the Celeſtial Bodies to be obſerved are neareſt to the Prime Vertical or eaſt and weſt Azimuth Circle, becauſe, then they keep neareſt the ſame Bearing, yet alter in Altitude faſteſt during ſmall Intervals of Time.

2. The Sextant or Octant, ſhould be well adjuſted, the Altitude accurately taken at the Time the Bearing is taken. The Compaſs ſhould be carefully tried before it is taken to Sea, by a Meridian Line accurately drawn, and at a proper Diſtance from all Iron or other Metal that can influence the Needle. In this, it ſhould agree with the beſt Needles at the ſame Place and Time; if theſe agree at Different Times, there may be no reaſon to ſuſpect Error in the Azimuth Compaſs at Sea; if they do not agree, the Error ſhould be allowed for at Sea, but, even then the Allowance may not give the true Variation.

3. Obſervations for the Variation, may be made by Altitudes of the Sun, Moon, Primary Planets, or Fixed Stars, as they occaſionally ſuit. If the Moon is obſerved out of the Meridian in an Oblique Sphere, the Bearing will be affected with the Effects of Parallax; to be accurate in this reſpect, her true Co-altitude, and Polar-diſtance ſhould be found, theſe with the Co-latitude, will give her true Azimuth (whilſt her obſerved Co-altitude, the Polar-diſtance, and the Co-latitude, will give her Azimuth as affected by Parallax). In the Torrid Zone, when the Moon is near the Prime Vertical, the Parallax in Altitude will be nearly this Correction in Altitude. When the Fixed Stars of the firſt Magnitude are applied, there will be frequent Opportunities of obſerving them at ſmall Altitudes.

4. The true and magnetic Azimuths or Bearings, being found and compared with each other, give the Variation, as in Section 66.

CONCLUSION.

1. From what has been delivered in this Work, it is evident that there are four practicable Methods of determining the Longitude of a Ship at Sea, and that when the Navigator is provided with the Inſtruments and Books belonging to them, they may be practiſed by him with Eaſe when he is at Sea.

2. The firſt Method, by keeping a Reckoning depends almoſt wholly on the Properties of Triangles and theſe depend on mathematical Principles; where this Method becomes erroneous through Imperfection in the Courſes and Diſtances eſtimated, Aſtronomy gives it its greateſt Step toward Perfection.

3. In the ſecond or Watch Method, the Time of one Meridian is carried to be compared with the Time of another Meridian, and their Sum or Difference is the Longitude. This Method depends on mechanical Principles for making the Watch, and on Aſtronomy when it is to do the Buſineſs for which it is deſigned.

4. The third or Lunar Method depends wholly on Aſtronomy, from the firſt Step taken in it to its Completion. Aſtronomy has for its Foundation all the mathematical Sciences in their moſt extenſive Applications, and the Laws of Matter and Motion in their greateſt Power and Perfection.

5. The fourth or Variation Method, depends chiefly on the Laws of Nature for thoſe of Magnetiſm, and on Aſtronomy in making it applicable for the Longitude at Sea.

6. It is therefore in the Power of almoſt every Navigator, to practiſe the firſt and third Methods, as they are ſo very eaſily to be learnt, and the Inſtruments belonging to them do not amount to a great Expence; and although the fourth Method never will be at all Times and Places applicable with equal Advantages; yet, if the Variation Lines are favourable, the Navigator ſhould never be unmindful of them, eſpecially at Places where ſtrong Currents ſet Ships toward the Coaſt.

ris's Place, the Latitude, and the Time at the Ship's Place are to be found; and thereby, the Longitude.

From what has been before delivered it is evident, that when the Method of finding the Longitude at Sea by the usual Method of Sailing, is practised without any other Aid than what can arise from its own Principles, it will ever be defective in long Voyages, and that its Defects, will, at all times and places, receive great help and assistance from Astronomy. That when the Method of finding the Longitude at Sea by the help of Watches or Time-keepers, is practised without any other Aid than what arises from its own Principles, no Inferences whatever can be drawn, either concerning the gaining or losing of the Time-keeper before it sets out on a Voyage, during its being at Sea, or when it comes to the End of the Voyage, without the application of Astronomy. That the Lunar Method, of finding the Longitude at Sea, whether it be by the Sun and Moon or the Moon and Zodiacal Stars, has for its Foundation, the Laws of the System of the World, the Principles and Practice of Astronomy; and therefore, it may be reasonably supposed, that the Principles on which this Method depends will always be invariably the same, and that the Practice of it can have no greater Difficulties than what arise in making the Observations and Calculations.

CL.

Instructions for finding the Longitude at Sea; by the help of Variation Charts.

1. In this Method, the Observer should be provided with a good Sextant or Octant for taking the Altitude of Sun, Moon, Planet, or Fixed Star, a good Azimuth Compass for taking the Magnetic Bearing of the Celestial Body, proper Assistants, and Books for making the Calculation. In this Method, the best times are when the Celestial Bodies to be observed are nearest to the Prime Vertical or east and west Azimuth Circle, because, then they keep nearest the same Bearing, yet alter in Altitude fastest during small Intervals of Time.

2. The Sextant or Octant, should be well adjusted, the Altitude accurately taken at the Time the Bearing is taken. The Compass should be carefully tried before it is taken to Sea, by a Meridian Line accurately drawn, and at a proper Distance from all Iron or other Metal that can influence the Needle. In this, it should agree with the best Needles at the same Place and Time; if these agree at Different Times, there may be no reason to suspect Error in the Azimuth Compass at Sea; if they do not agree, the Error should be allowed for at Sea, but, even then the Allowance may not give the true Variation.

3. Observations for the Variation, may be made by Altitudes of the Sun, Moon, Primary Planets, or Fixed Stars, as they occasionally suit. If the Moon is observed out of the Meridian in an Oblique Sphere, the Bearing will be affected with the Effects of Parallax; to be accurate in this respect, her true Co-altitude, and Polar-distance should be found, these with the Co-latitude, will give her true Azimuth (whilst her observed Co-altitude, the Polar-distance, and the Co-latitude, will give her Azimuth as affected by Parallax). In the Torrid Zone, when the Moon is near the Prime Vertical, the Parallax in Altitude will be nearly this Correction in Altitude. When the Fixed Stars of the first Magnitude are applied, there will be frequent Opportunities of observing them at small Altitudes.

4. The true and magnetic Azimuths or Bearings, being found and compared with each other, give the Variation, as in Section 66.

CONCLUSION.

1. From what has been delivered in this Work, it is evident that there are four practicable Methods of determining the Longitude of a Ship at Sea, and that when the Navigator is provided with the Instruments and Books belonging to them, they may be practised by him with Ease when he is at Sea.

2. The first Method, by keeping a Reckoning depends almost wholly on the Properties of Triangles and these depend on mathematical Principles; where this Method becomes erroneous through Imperfection in the Courses and Distances estimated, Astronomy gives it its greatest Step toward Perfection.

3. In the second or Watch Method, the Time of one Meridian is carried to be compared with the Time of another Meridian, and their Sum or Difference is the Longitude. This Method depends on mechanical Principles for making the Watch, and on Astronomy when it is to do the Business for which it is designed.

4. The third or Lunar Method depends wholly on Astronomy, from the first Step taken in it to its Completion. Astronomy has for its Foundation all the mathematical Sciences in their most extensive Applications, and the Laws of Matter and Motion in their greatest Power and Perfection.

5. The fourth or Variation Method, depends chiefly on the Laws of Nature for those of Magnetism, and on Astronomy in making it applicable for the Longitude at Sea.

6. It is therefore in the Power of almost every Navigator, to practise the first and third Methods, as they are so very easily to be learnt, and the Instruments belonging to them do not amount to a great Expence; and although the fourth Method never will be at all Times and Places applicable with equal Advantages; yet, if the Variation Lines are favourable, the Navigator should never be unmindful of them, especially at Places where strong Currents set Ships toward the Coast.

XCI.

☞ In Section 142 of this Work, is a Method whereby the Meridian Transits of the Celestial Bodies may be taken without erring more than two or three Seconds of Time. If this Method is applied for getting the Times of the Meridian Transits of the Sun, Moon, Primary Planets and Fixed Stars, and the Times are observed by help of a Pendulum Clock set as accurately as possible to mean Solar Time, such Observations may be applied not only to the Corrections of the geographical Longitudes of Places at great Distances from each other, but also to the Discovery of small Errors in Ephemerides or the original Tables from which they be made. In this Method, the Lines must be as near the Plane of the Meridian as possible, the Observations must be taken as correctly as possible, and the daily Change in Right Ascension must be duely allowed for, both in deducing the true Places of the Celestial Bodies and in inferring the Difference of Longitude by such correspondent Observations. The Particulars to be observed, in order to come to the forementioned Accuracy, may be seen in the Treatise on Practical Astronomy.

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